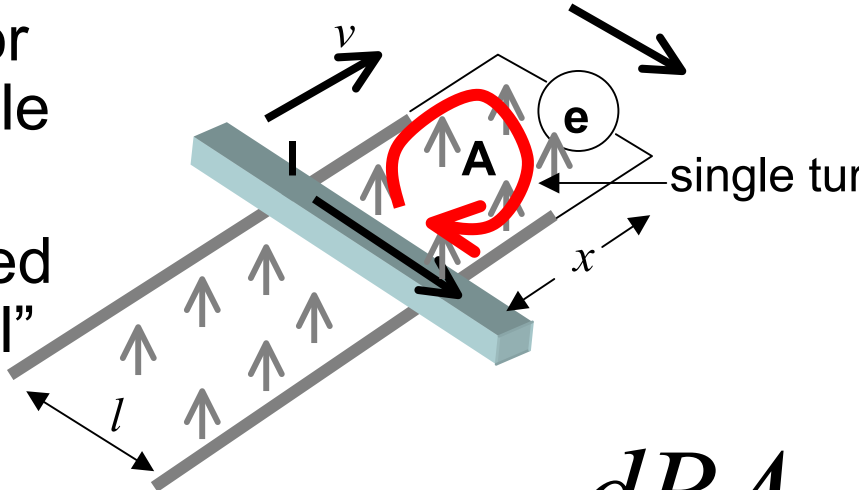
19222 Electrical Machines and Control

REAL Transformers

Linear Generator

- Recall that the linear generator could be considered as a single turn of wire in a magnetic flux
- The magnetic flux was changed by altering the area of the “coil” – area A
- This induced an emf (voltage) across the rails and caused a current to flow in the sliding contact which *opposed* the force causing the movement
- Could achieve same affect with stationary slider and varying the flux density



$$e = N\frac{dBA}{dt}$$

$$e = N\frac{d\Phi}{dt}$$

Self Inductance

- Recall coil wrapped round core induced a flux
- Flux passing through coil induces an emf – self inductance

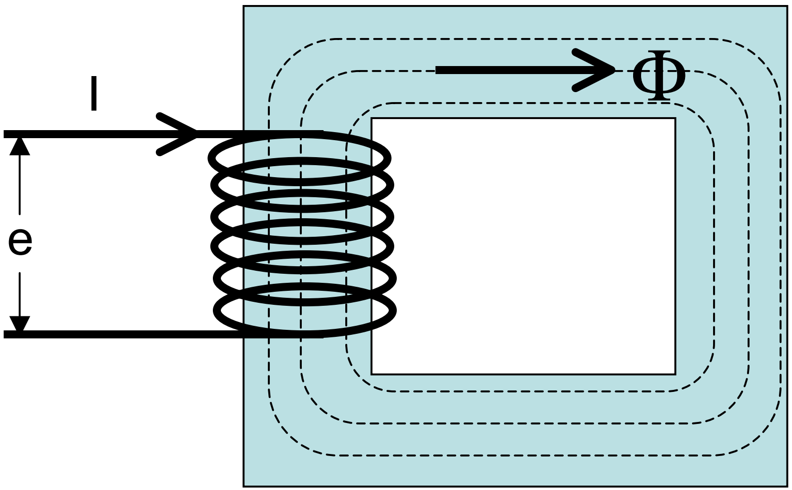
$$\Phi = \frac{NI}{\Re}$$

$$e = N\frac{d\Phi}{dt} = N\frac{d\left(\frac{NI}{\Re}\right)}{dt}$$

$$e = \frac{N^2}{\Re}\frac{dI}{dt}$$

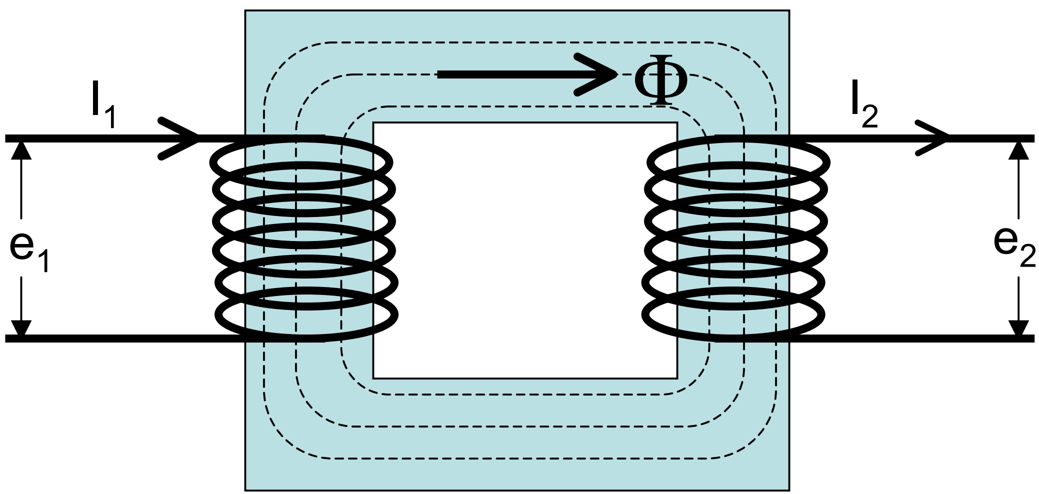
recall

$$e = L\frac{dI}{dt} \qquad L = \frac{N^2}{\Re}$$



Mutual Inductance

- Consider two coils wrapped round the same core
- Flux Φ generated by coil 1 “links” with coil 2 and generates an emf e_2



EMF(Voltage) Relationship

- The relationships between the two emfs (voltages) is:

$$e_2 = N_2 \frac{d\Phi}{dt} \qquad \Phi = \frac{N_1 I_1}{\Re}$$

$$e_2 = \frac{N_1 N_2}{\Re} \frac{dI_1}{dt} \Rightarrow \frac{e_2}{N_2} = \frac{N_1}{\Re} \frac{dI_1}{dt}$$

recall

$$e_1 = \frac{N_1 N_1}{\Re} \frac{dI_1}{dt} \Rightarrow \frac{e_1}{N_1} = \frac{N_1}{\Re} \frac{dI_1}{dt}$$

by inspection

$$\frac{e_1}{N_1} = \frac{e_2}{N_2} \Rightarrow \frac{e_1}{e_2} = \frac{N_1}{N_2}$$

AC Operation

- Notice that the expressions for e_1 and e_2 include *dI/dt*
- For e_1 and $e_2 \neq 0$ time varying current is required and e_1 and e_2 will vary sinusoidally
- Flux therefore also varies sinusoidally*

$$\frac{e_2}{N_2} = \frac{e_1}{N_1} = \frac{N_1}{\Re}\frac{dI_1}{dt}$$

$$e = N\frac{d\Phi}{dt}$$

$$e(t) = \hat{E}\sin(\omega t)$$

$$\Phi = \frac{1}{N}\int_0^t \hat{E}\sin(\omega t)$$

$$\Phi = -\frac{1}{\omega N}\hat{E}\cos(\omega t)$$

$$\Phi = -\hat{\Phi}\cos(\omega t)$$

AC Operation

- Can use phasor quantities with transformer equations so:

$$\frac{\mathbf{e}_1}{\mathbf{e}_2} = \frac{|\mathbf{e}_1|}{|\mathbf{e}_2|} = \frac{N_1}{N_2} \qquad \mathbf{e} = e\angle\theta$$

- Assuming an energy balance in the transformer:

$$|\mathbf{S}_{in}| = |\mathbf{S}_{out}| \Rightarrow |\mathbf{e}_1||\mathbf{I}_1| = |\mathbf{e}_2||\mathbf{I}_2|$$

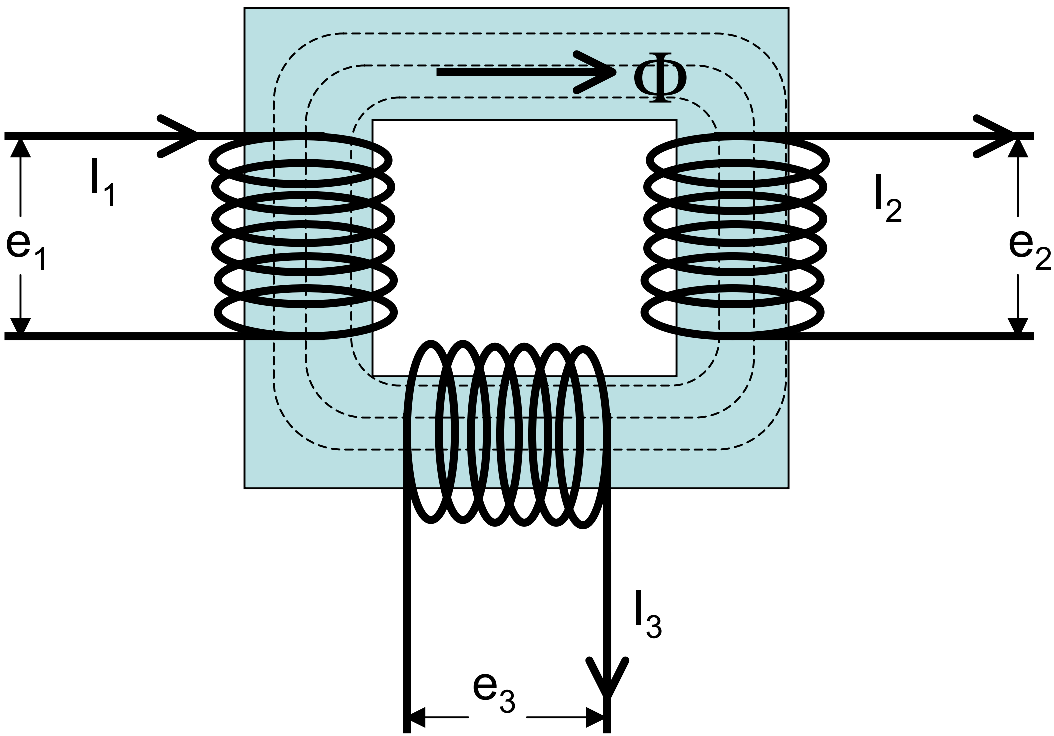
$$\frac{|\mathbf{e}_1|}{|\mathbf{e}_2|} = \frac{|\mathbf{I}_2|}{|\mathbf{I}_1|} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} \qquad \mathbf{I} = I\angle\phi$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$

Multiple Windings

- For multiple windings:

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2 + N_3\mathbf{I}_3 + \ldots + N_n\mathbf{I}_n \quad \text{OR} \quad \sum \mathbf{F} = 0$$



Multiple Windings

- Similarly for emf (voltages):

$$\frac{N_1}{N_2} = \frac{\mathbf{e}_1}{\mathbf{e}_2}$$

$$\frac{N_1}{N_3} = \frac{\mathbf{e}_1}{\mathbf{e}_3}$$

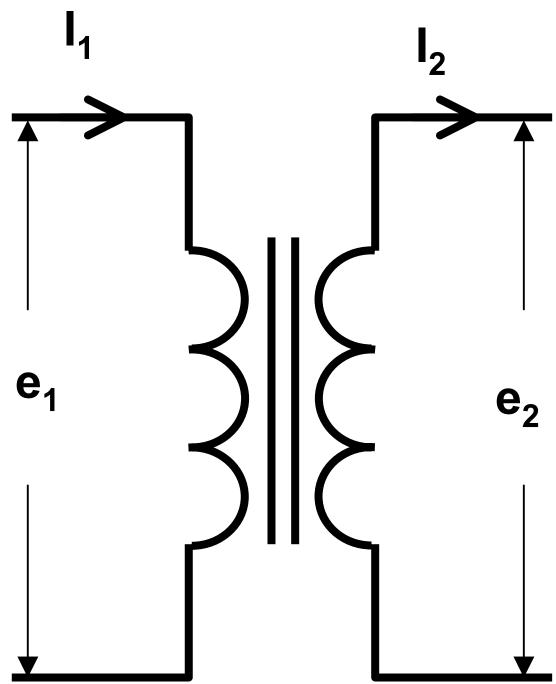
$$\frac{N_2}{N_3} = \frac{\mathbf{e}_2}{\mathbf{e}_3}$$

Ideal Transformer

- Basic equations:

$$\frac{N_1}{N_2} = \frac{\mathbf{e}_1}{\mathbf{e}_2}$$

$$N_1 \mathbf{I}_1 = N_2 \mathbf{I}_2$$



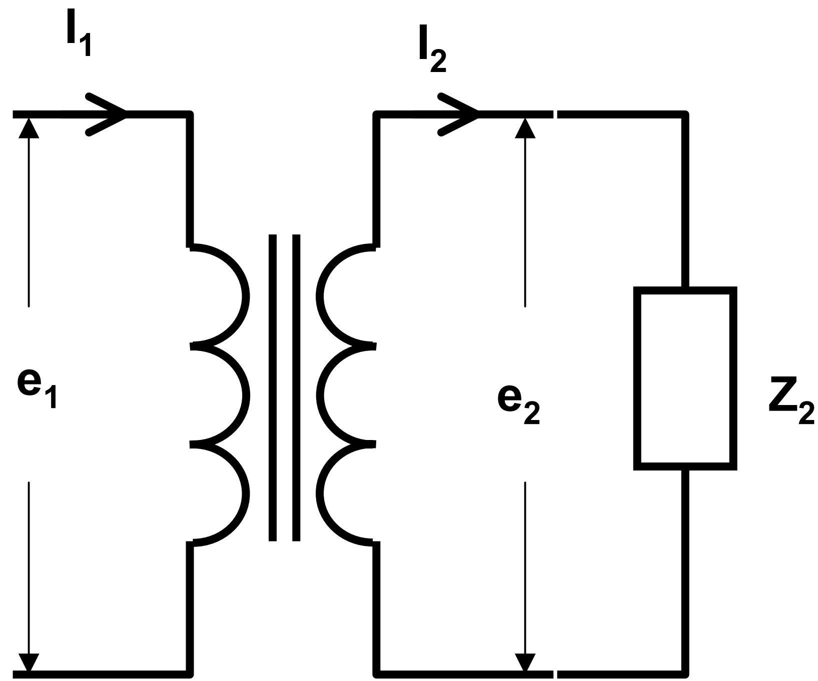
Ideal Transformer Under Load

- Load can be "reflected" to primary side

$$Z_2 = \frac{\mathbf{e}_2}{\mathbf{I}_2} \quad \mathbf{e}_1 = \frac{N_1}{N_2}\mathbf{e}_2$$

$$\mathbf{I}_1 = \frac{N_2}{N_1}\mathbf{I}_2$$

$$Z_1 = \frac{\mathbf{e}_1}{\mathbf{I}_1} = \frac{\frac{N_1}{N_2}\mathbf{e}_2}{\frac{N_2}{N_1}\mathbf{I}_2} = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

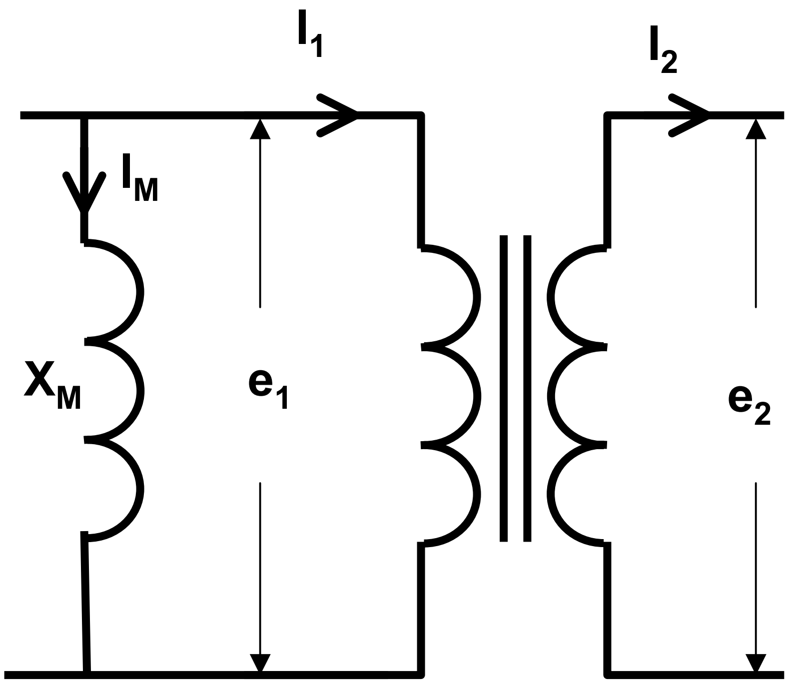


Non Ideal Transformer

- For magnetic flux to flow

$$\mathbf{F}_1 > \mathbf{F}_2$$

$$N_1 \mathbf{I}_1 > N_2 \mathbf{I}_2$$



- Add a "magnetisation current"

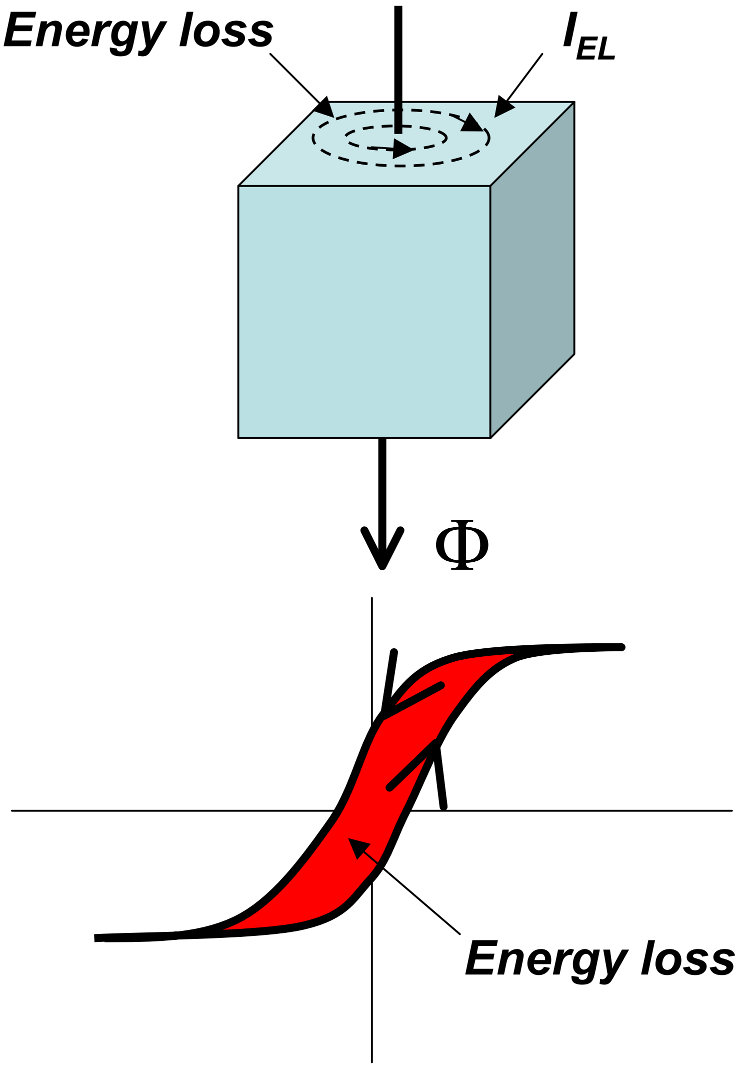
$$N_1 \mathbf{I}_1 > N_2 \mathbf{I}_2$$

$$N_1 \mathbf{I}_1 = N_2 \mathbf{I}_2 + \mathbf{I}_M$$

Non Ideal Transformer

- Eddy Losses
- Hysteresis losses
- Both energy losses – commonly called “iron losses”:

$$P_{loss} = I_{Fe}^{2} R_{Fe}$$

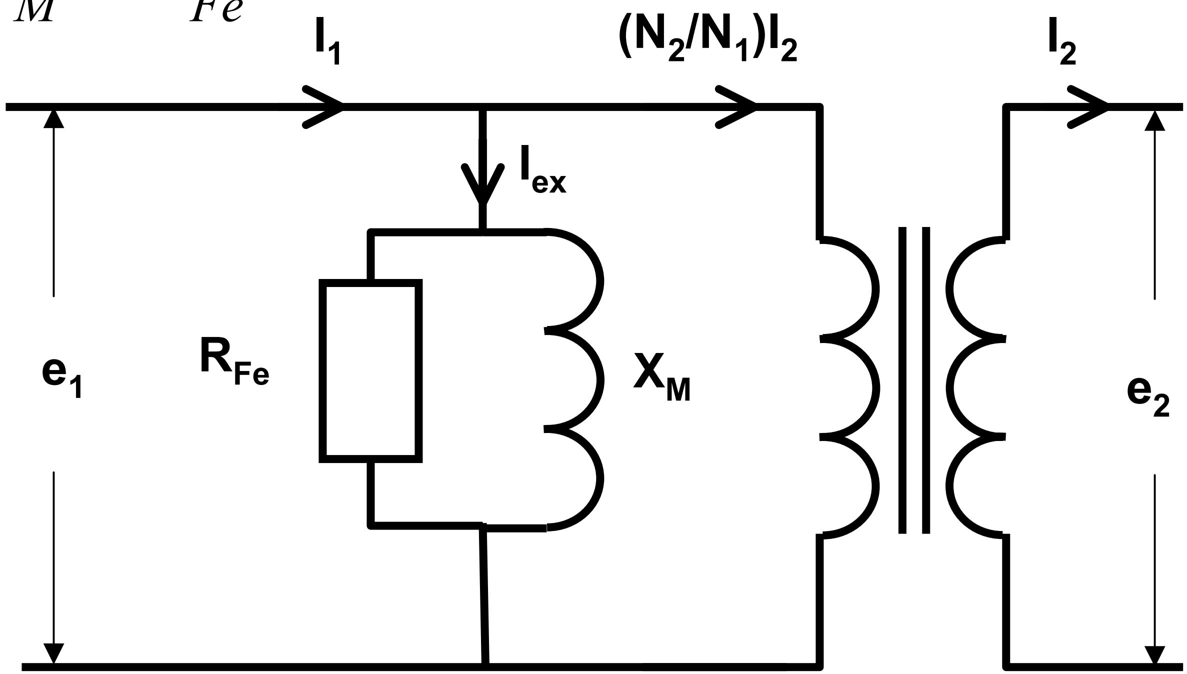


Non Ideal Transformer

- Add to "magnetisation current"

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2 + \mathbf{I}_M + \mathbf{I}_{Fe}$$

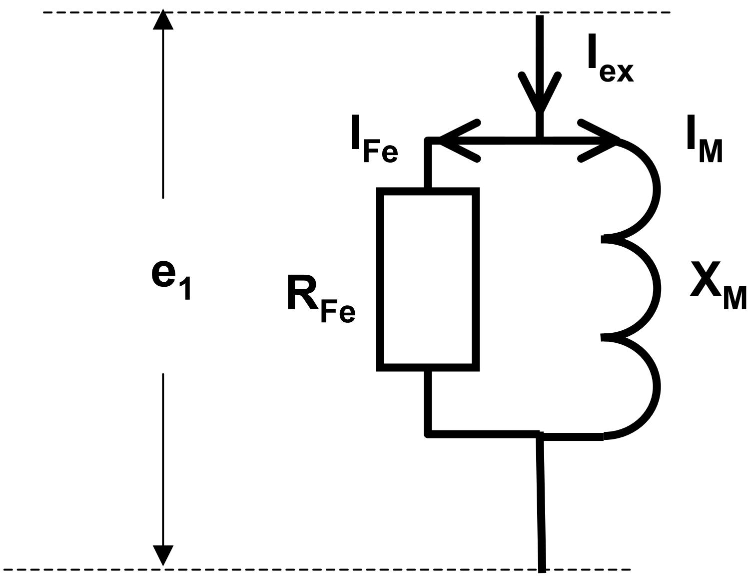
$$\mathbf{I}_{ex} = \mathbf{I}_M + \mathbf{I}_{Fe}$$



Non Ideal Transformer

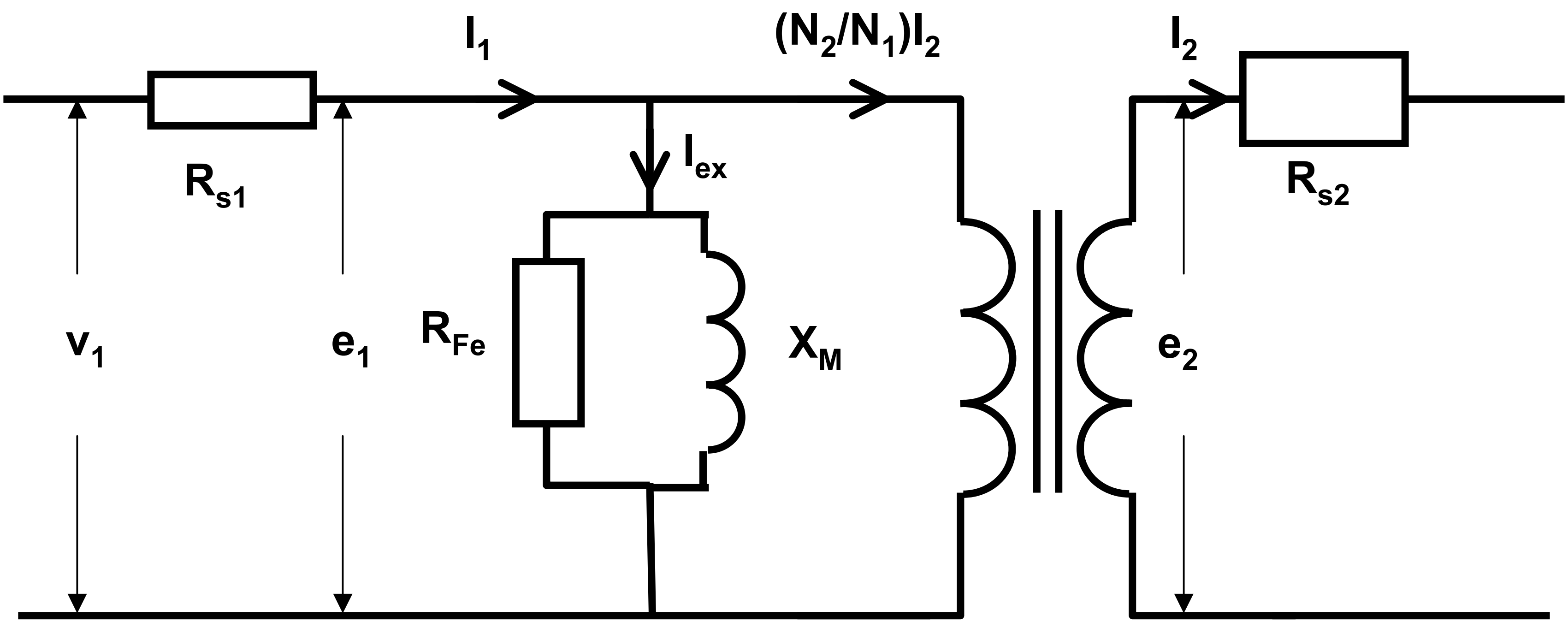
- The excitation current can be calculated from:

$$\mathbf{I}_{ex} = \frac{\mathbf{e}_1}{R_{Fe}} - j\frac{\mathbf{e}_1}{X_M}$$



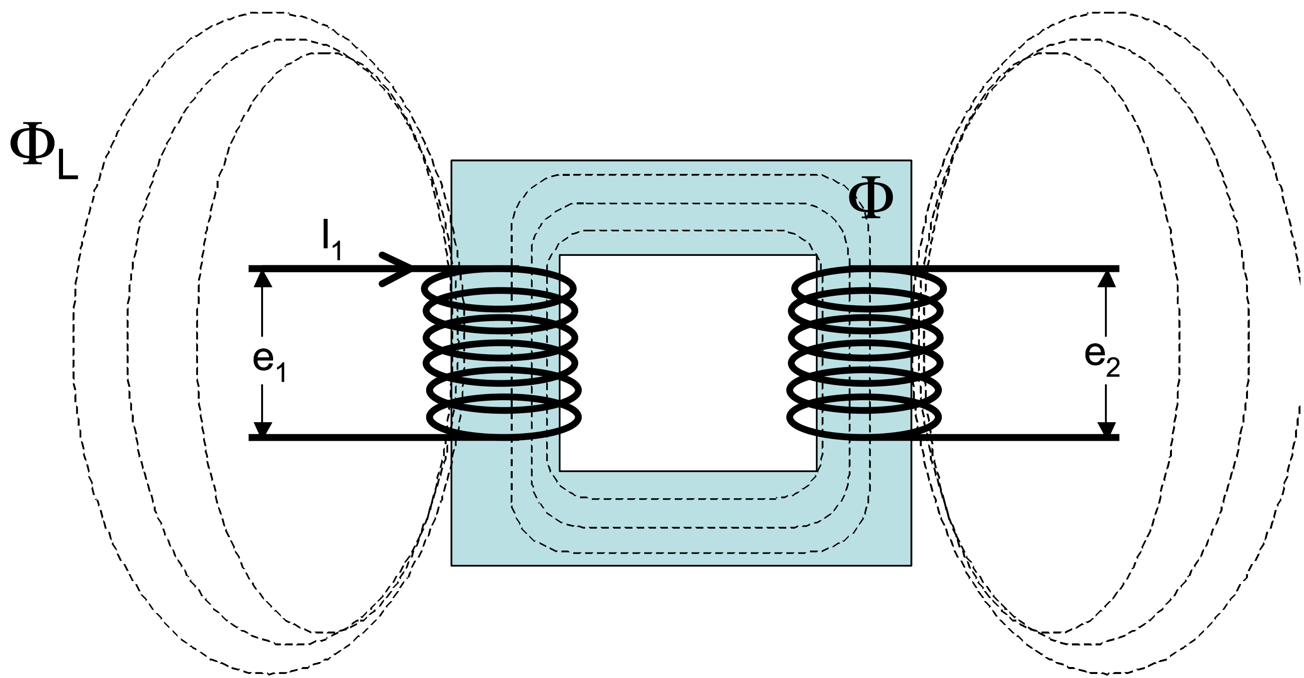
Non Ideal Transformer

- The windings of the transformer have some resistance – known as “copper losses”
- Add resistors in series to account for this:

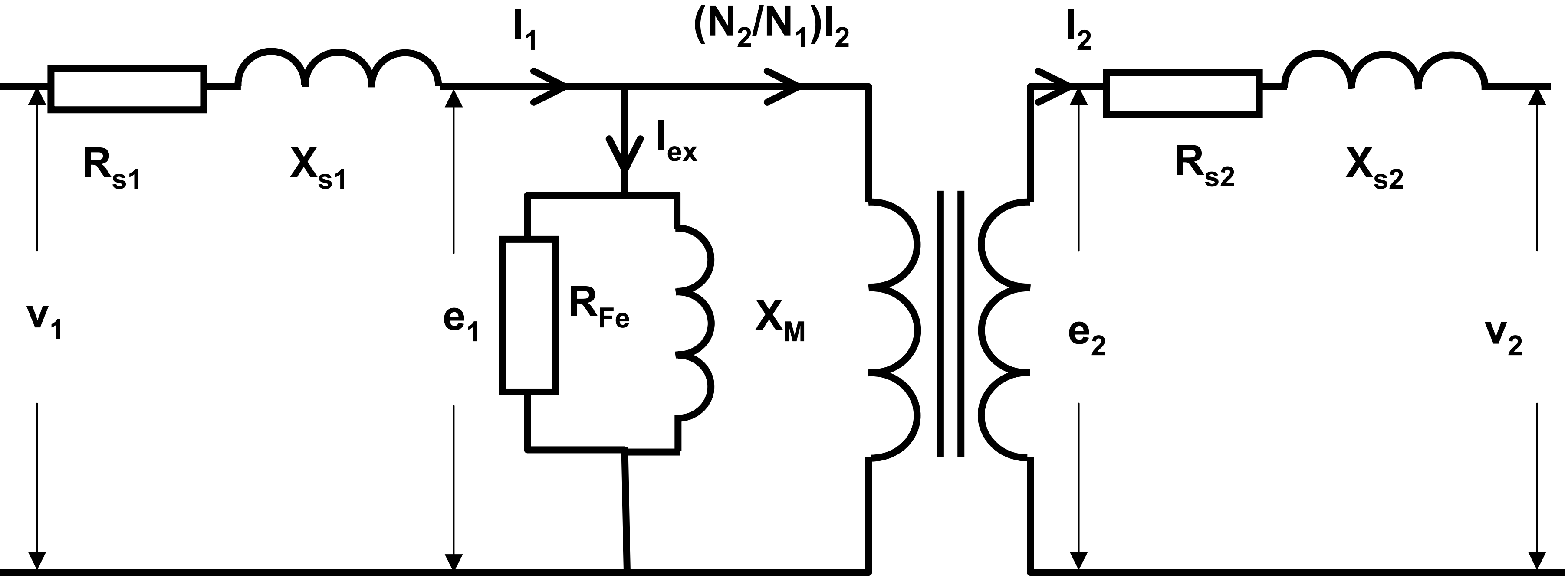


Non Ideal Transformer

- Leakage flux – not all the magnetic flux from the primary travels round the core
- Add series inductances to model

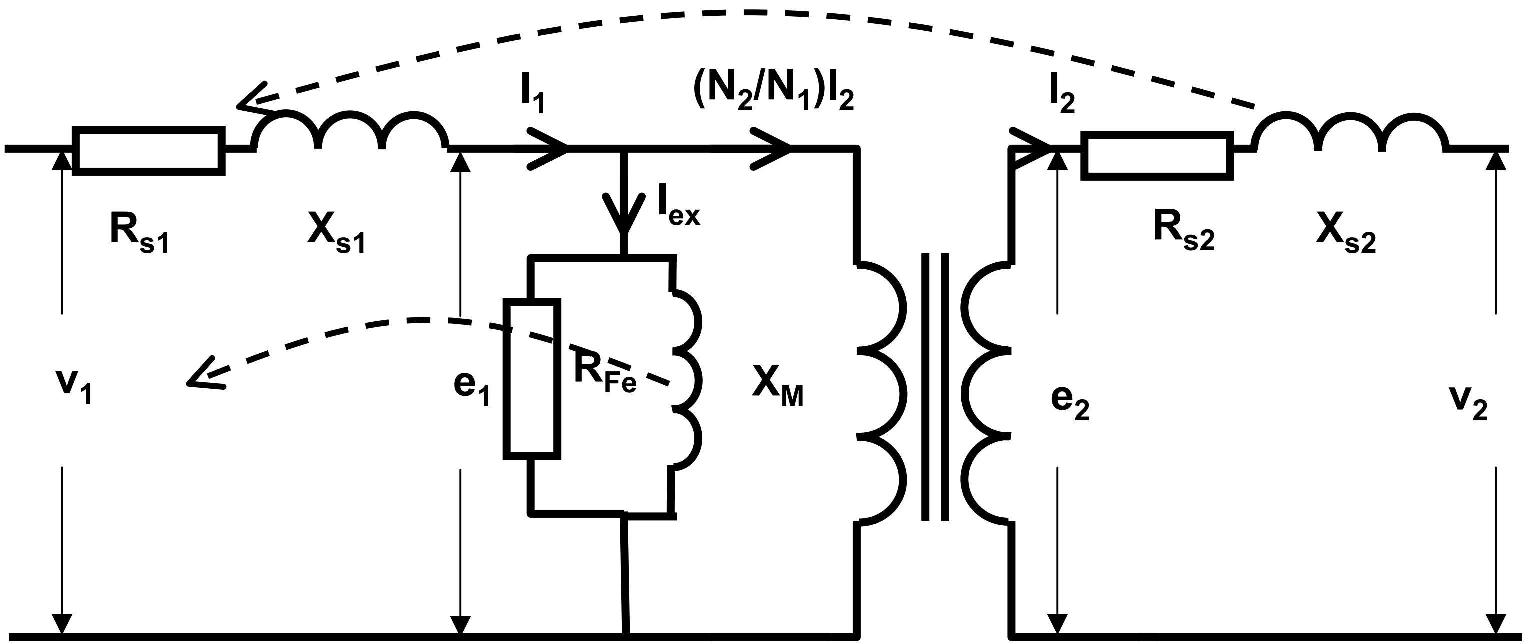


Non Ideal Transformer



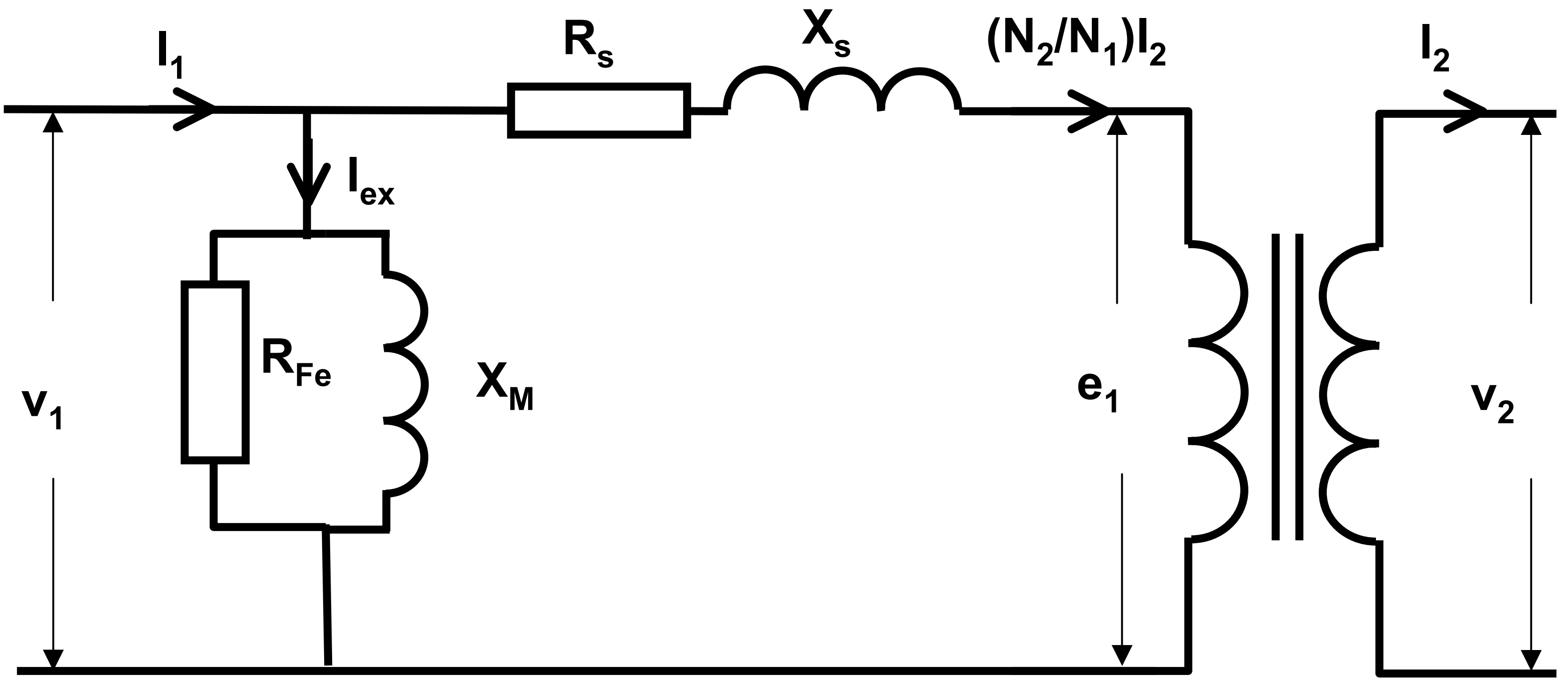
Simplification

- Can simplify model slightly – move secondary impedance to primary + move excitation branch (minimal error if R_{s1} and X_{s1} are small)



Simplified Model

$$X_s = X_{s1} + \left(\frac{N_1}{N_2}\right)^2 X_{s2} \qquad R_s = R_{s1} + \left(\frac{N_1}{N_2}\right)^2 R_{s2}$$



Parameter Tests

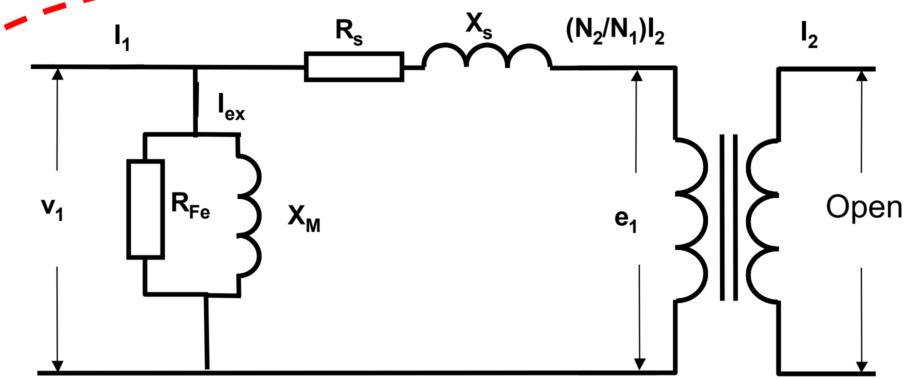
- Open circuit for R_{fe} , $I_2 = 0$

$$R_{Fe} = \frac{\mathbf{V}_1^{\,2}}{P_{in}}$$

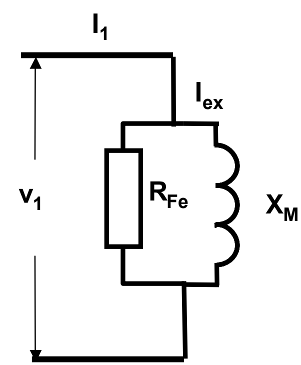
$$X_M = \frac{\mathbf{V}_1^{\,2}}{Q_{in}}$$

$$= \frac{\mathbf{V}_1^{\,2}}{\sqrt{|\mathbf{S}_{in}|^2 - |P_{in}|^2}}$$

$$|\mathbf{S}_{in}| = |\mathbf{V}_1||\mathbf{I}_1|$$



Simplifies to



Parameter Tests

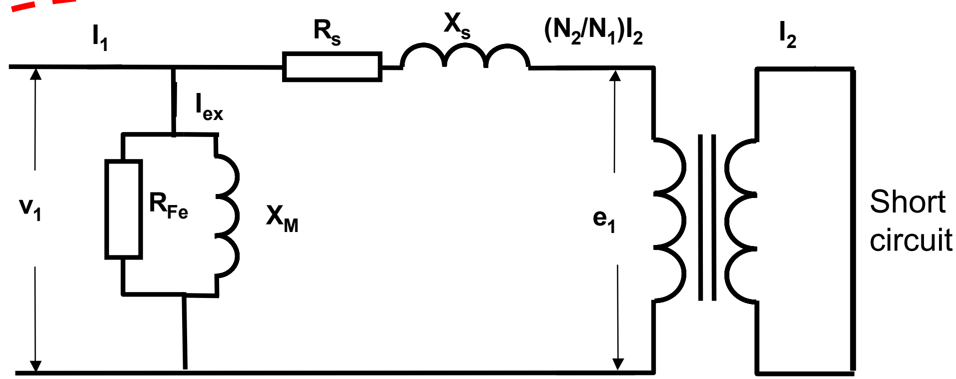
- Short circuit for R_{fe} , $V_1 \rightarrow 0$; $I_{ex} \rightarrow 0$

$$R_S = \frac{P_{in}}{|\mathbf{I}_1|^2}$$

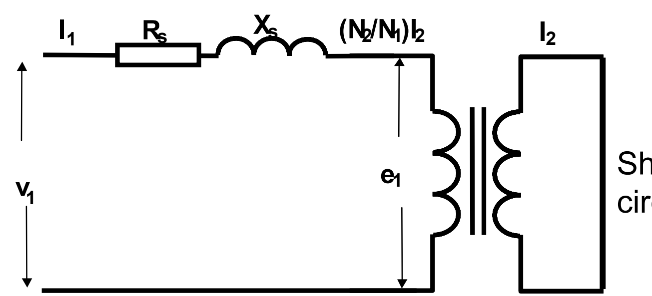
$$X_S = \frac{Q_{in}}{|\mathbf{I}_1{}^2|}$$

$$= \frac{\sqrt{|\mathbf{S}_{in}|^2 - |P_{in}|^2}}{|\mathbf{I}_1{}^2|}$$

$$|\mathbf{S}_{in}| = |\mathbf{V}_1||\mathbf{I}_1|$$



Simplifies to



Regulation

- Copper losses decrease output voltage as current (load) increases

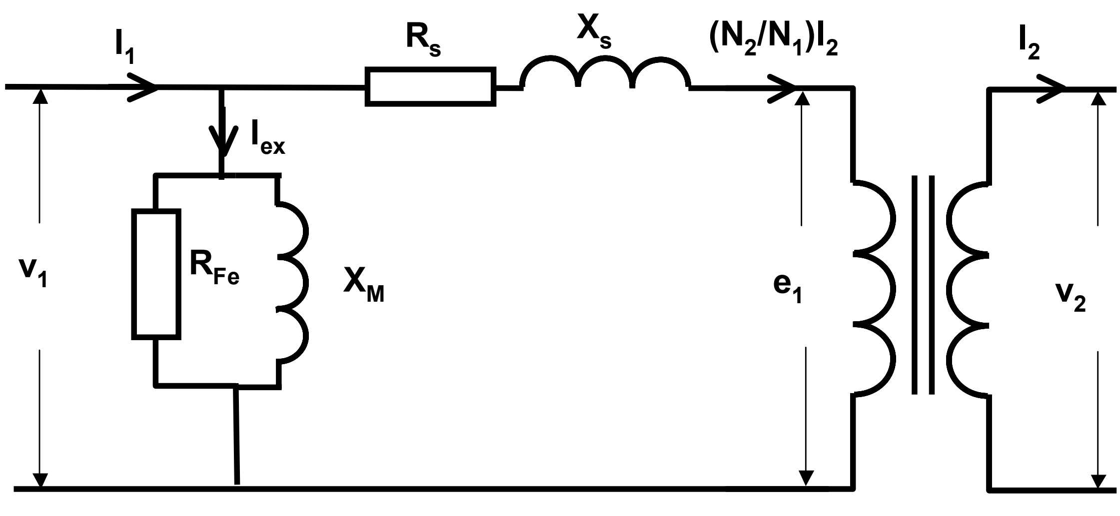
$$\text{Regulation} = \frac{V_{no\ load} - V_{load}}{V_{no\ load}} \times 100$$

$$\text{Regulation} = \frac{V_{no\ load} - V_{load}}{V_{no\ load}} \times 100$$

$$\mathbf{V}_{no\ load} = \frac{N_2}{N_1}\mathbf{V}_1$$

$$\mathbf{V}_{load} = \frac{N_2}{N_1}\mathbf{e}_1$$

$$\mathbf{e}_1 = \mathbf{V}_1 - \frac{N_2}{N_1}\mathbf{I}_2(R_s + jX_s)$$

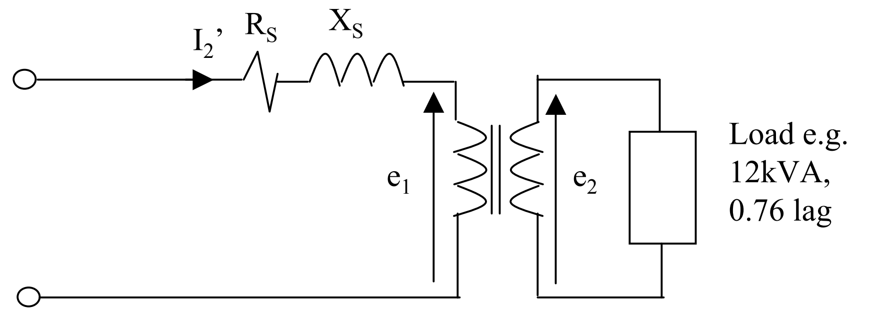


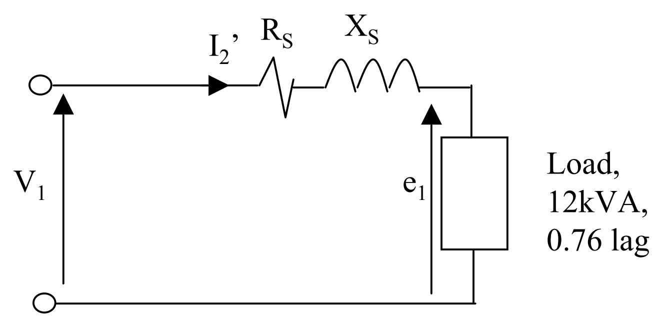
Regulation (contd)

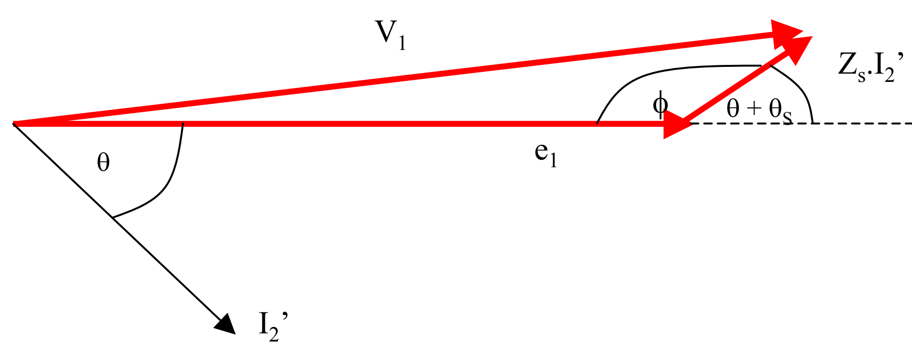
- Load can be referred to primary
- Phasor diagram constructed where θ_S is the angle of $R_s + jX_s$
- θ is from the power factor taking e_1 as ref.
- From $S = |e_1|.|I_2'|$ and

$$|V_1|^2 = |e_1|^2 + \left(|Z_s.I_2'|\right)^2 - 2.|e_1|.|Z_s.I_2'|.\cos(\phi)$$

solve for e_1 and then for regulation







Efficiency

- The efficiency of the transformer can be calculated as follows

$$P_{out} = I_2 V_2 \cos\phi$$

$$P_{loss} = \frac{V_1^{\,2}}{R_{Fe}} + \left(\frac{N_2}{N_1} \left| \mathbf{I}_2 \right| \right)^2 R_s$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}}$$