

THE PER-UNIT SYSTEM

An interconnected power system typically consists of many different voltage levels given a system containing several transformers and/or rotating machines. The *per-unit system* simplifies the analysis of complex power systems by choosing a common set of base parameters in terms of which, all systems quantities are defined. The different voltage levels disappear and the overall system reduces to a set of impedances. The primary advantages of the per-unit system are:

- (1) The per-unit values for transformer impedance, voltage and current are identical when referred to the primary and secondary (no need to reflect impedances from one side of the transformer to the other, the transformer is a single impedance).
- (2) The per-unit values for various components lie within a narrow range regardless of the equipment rating.
- (3) The per-unit values clearly represent the relative values of the circuit quantities. Many of the ubiquitous scaling constants are eliminated.
- (4) Ideal for computer simulations.

The definition of any quantity (voltage, current, power, impedance) in the per-unit system is

$$\text{Quantity (per unit)} = \frac{\text{Quantity (normal units)}}{\text{Base value of quantity (normal units)}}$$

The complete characterization of a per-unit system requires that all four base values be defined. Given the four base values, the per-unit quantities are defined as

$$V_{pu} = \frac{V}{V_{base}} \quad I_{pu} = \frac{I}{I_{base}} \quad S_{pu} = \frac{S}{S_{base}} \quad Z_{pu} = \frac{Z}{Z_{base}}$$

Note that the numerator terms in the previous equations are in general complex while the base values are real-valued. Once any two of the four base values (V_{base} , I_{base} , S_{base} , and Z_{base}) are defined, the remaining two base values can be determined according their fundamental circuit relationships. Usually the base values of power and voltage are selected and the base values of current and impedance are determined according to

$$I_{base} = \frac{S_{base}}{V_{base}}$$
$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^2}{S_{base}}$$

If we also assume that:

- (1) the value of S_{base} is constant for all points in the power system, and
- (2) the ratio of voltage bases on either side of a transformer is chosen to equal the ratio of the transformer voltage ratings,

then the transformer per-unit impedance remains unchanged when referred from one side of a transformer to the other. This will allow us to eliminate the ideal transformer from the transformer model (i.e., we will not have to reflect impedances from one side of the transformer to the other.)

Example (Per-unit system, transformer models)

A single phase 20 kVA, 480/120, 60 Hz single-phase transformer has an impedance of $\mathbf{Z}_{eq2} = 0.0525 \angle 78.13^\circ \Omega$ referred to the LV winding. Determine the per-unit transformer impedance referred to the LV winding and the HV winding.

The transformer impedance referred to the HV winding is given by

$$\mathbf{Z}_{eq1} = a^2 \mathbf{Z}_{eq2} = \left(\frac{480}{120} \right)^2 (0.0525 \angle 78.13^\circ) = 0.84 \angle 78.13^\circ \Omega$$

According to our convention, the base values for this system are

$$S_{base} = 20 \text{ kVA} \quad V_{base1} = 480 \text{ V} \quad V_{base2} = 120 \text{ V}$$

The resulting impedance bases are

$$Z_{base1} = \frac{V_{base1}^2}{S_{base}} = \frac{480^2}{20000} = 11.52 \Omega$$

$$Z_{base2} = \frac{V_{base2}^2}{S_{base}} = \frac{120^2}{20000} = 0.72 \Omega$$

and the resulting per-unit impedances are

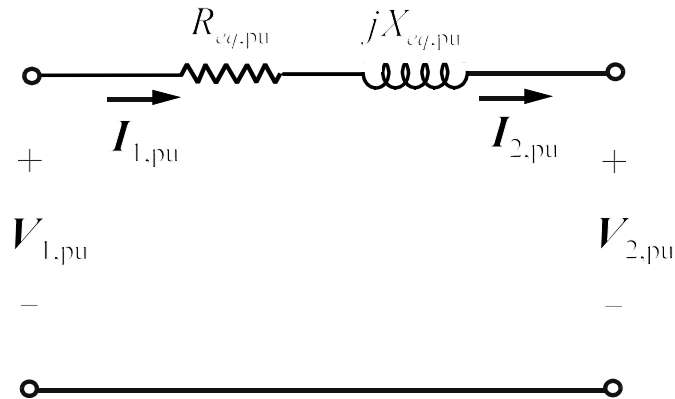
$$\mathbf{Z}_{eq1,pu} = \frac{\mathbf{Z}_{eq1}}{Z_{base1}} = \frac{0.84 \angle 78.13^\circ}{11.52} = 0.0729 \angle 78.13^\circ \text{ pu}$$

$$\mathbf{Z}_{eq2,pu} = \frac{\mathbf{Z}_{eq2}}{Z_{base2}} = \frac{0.0525 \angle 78.13^\circ}{0.72} = 0.0729 \angle 78.13^\circ \text{ pu}$$

Since the per-unit impedances are equal for both sides of the transformer, we may designate a single transformer impedance.

$$\mathbf{Z}_{eq,pu} = \mathbf{Z}_{eq1,pu} = \mathbf{Z}_{eq2,pu}$$

Given that the per-unit transformer impedances referred to the two sides of the transformer are equal, the per-unit equivalent model for the transformer reduces to that shown below.



Note that there are no primed quantities in the per-unit equivalent circuit. Thus, there is no need to reflect impedances when using this model. Using the per-unit transformer model eliminates the need to scale quantities by the transformer turns ratio, thus eliminating a common source for error.

The per-unit full-load copper loss associated with a given transformer can be shown to be equal to $R_{eq,pu}$. According to the previously given definition of transformer copper loss,

$$P_{Cu} = I_1^2 R_{eq1} = I_2^2 R_{eq2}$$

the per-unit copper loss is given by

$$P_{Cu,pu} = \frac{I_1^2 R_{eq1}}{S_{base}} = \frac{I_2^2 R_{eq2}}{S_{base}}$$

where $S_{base} = V_{1,rated} I_{1,rated} = V_{2,rated} I_{2,rated}$. Under full-load conditions, the primary and secondary current and voltage magnitudes are the transformer rated values. This gives

$$\begin{aligned}
P_{Cu,pu}^{FL} &= \frac{I_{1,rated}^2 R_{eq1}}{V_{1,rated} I_{1,rated}} = \frac{I_{2,rated}^2 R_{eq2}}{V_{2,rated} I_{2,rated}} \\
&= \frac{R_{eq1}}{V_{1,rated}/I_{1,rated}} = \frac{R_{eq2}}{V_{2,rated}/I_{2,rated}} \\
&= \frac{R_{eq1}}{Z_{base1}} = \frac{R_{eq2}}{Z_{base2}} \\
&= R_{eq,pu}
\end{aligned}$$

Thus, the resistance term in the per-unit transformer equivalent circuit also represents the per-unit value of the transformer full-load copper loss.

Example (Per-unit transformer equivalent circuit)

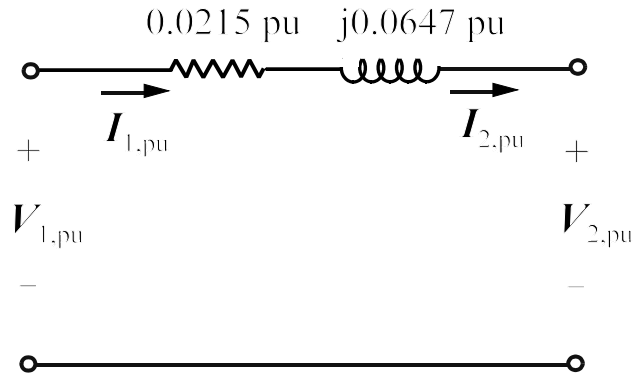
The equivalent impedance of a 10 kVA, 2200 V/220 V, 60 Hz transformer is $10.4 + j31.3 \Omega$ when referred to the high-voltage side. The transformer core losses are 120 W. Determine (a.) the per-unit equivalent circuit (b.) the voltage regulation when the transformer delivers 75% of full load at a power factor of 0.6 lagging, and (c.) the transformer efficiency given the transformer load of part (b.).

$$(a.) \quad S_{base} = 10 \text{ kVA} \quad V_{base1} = 2200 \text{ V} \quad V_{base2} = 220 \text{ V}$$

$$Z_{base1} = \frac{V_{base1}^2}{S_{base}} = \frac{2200^2}{10000} = 484 \Omega$$

$$Z_{base2} = \frac{V_{base2}^2}{S_{base}} = \frac{220^2}{10000} = 4.84 \Omega$$

$$Z_{eq,pu} = \frac{Z_{eq1}}{Z_{base1}} = \frac{10.4 + j31.3}{484} = 0.0215 + j0.0647 \text{ pu}$$



- (b.) For a load which is 75% of rated value at a power factor of 0.6 lagging ($\theta_v - \theta_i = 53.13^\circ$), the per-unit values of the voltage $V_{2,pu}$ and the current $I_{2,pu}$ are (using the voltage $V_{2,pu}$ as the phase reference)

$$V_{2,pu} = 1 \angle 0^\circ \text{ pu} \quad I_{2,pu} = 0.75 \angle -53.13^\circ \text{ pu}$$

$$\begin{aligned} V_{1,pu} &= V_{2,pu} + I_{2,pu} Z_{eq,pu} \\ &= 1 \angle 0^\circ + (0.75 \angle -53.13^\circ)(0.0682 \angle 71.62^\circ) \\ &= 1 \angle 0^\circ + 0.0512 \angle 18.49^\circ \\ &= 1.0487 \angle 0.89^\circ \end{aligned}$$

$$\begin{aligned} VR &= \frac{|V_{1,pu}| - |V_{2,pu}|}{|V_{2,pu}|} \times 100 \\ &= \frac{1.0487 - 1}{1} \times 100 \\ &= 4.87\% \end{aligned}$$

- (c.) The transformer efficiency in terms of per-unit values is

$$\eta = \frac{P_{out,pu}}{P_{out,pu} + P_{core,pu} + P_{Cu,pu}} \times 100$$

The transformer per-unit real output power is found by multiplying the per-unit complex output power by the power factor.

$$S_{out,pu} = S_{2,pu} = V_{2,pu} I_{2,pu} = (1)(0.75) = 0.75 \text{ pu}$$

$$P_{out,pu} = P_{2,pu} = S_{2,pu} PF = (0.75)(0.6) = 0.45 \text{ pu}$$

The full load per-unit copper losses are equal to the real part of the per-unit transformer impedance.

$$P_{Cu,pu}^{FL} = R_{eq,pu} = 0.0215 \text{ pu}$$

This relationship can be stated in terms of the current and resistance under full load conditions.

$$P_{Cu,pu}^{FL} = (I_{2,pu}^{FL})^2 R_{eq,pu} = (1)^2(0.0215) = 0.0215 \text{ pu}$$

At 75% of the rated current, the per-unit copper losses are

$$P_{Cu,pu} = (I_{2,pu})^2 R_{eq,pu} = (0.75)^2(0.0215) = 0.0121 \text{ pu}$$

The per-unit core losses are found by normalizing the actual core losses.

$$P_{core,pu} = \frac{P_{core}}{S_{base}} = \frac{120}{10000} = 0.012 \text{ pu}$$

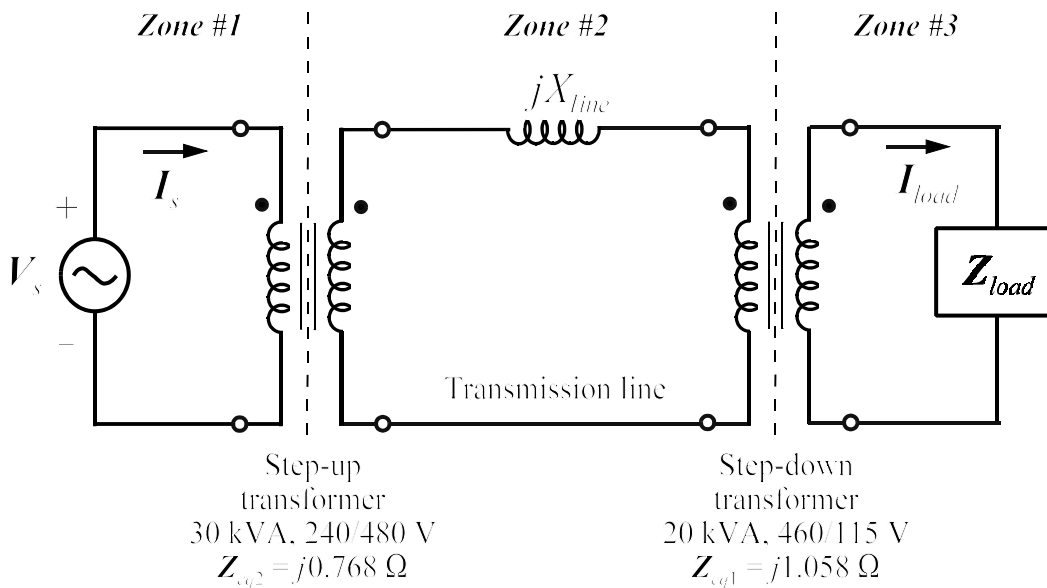
The efficiency of the transformer is

$$\begin{aligned} \eta &= \frac{0.45}{0.45 + 0.012 + 0.0121} \times 100 \\ &= 94.92\% \end{aligned}$$

Example (System analysis using per-unit model)

Given $V_s = 220 \angle 0^\circ$, $X_{line} = 2 \Omega$ and $Z_{load} = (0.9 + j0.2) \Omega$ in the two transformer circuit shown below,

- draw the per-unit circuit, labeling all values.
- determine the per-unit load current and the actual load current.



We start by choosing S_{base} . With more than one transformer in the circuit, we could choose either kVA rating. Let $S_{base} = 30$ kVA. The value of V_{base} should be chosen in the three zones to match the ratio of the voltage ratings on either side of each transformer. If we choose

$$V_{base1} = 240 \text{ V} \quad V_{base2} = 480 \text{ V}$$

in order to match the voltage ratings of the step-up transformer, we must choose V_{base3} such that the ratio of V_{base2} to V_{base3} equals the voltage ratio of 460/115 or

$$\frac{V_{base2}}{V_{base3}} = \frac{460}{115} \Rightarrow V_{base3} = \frac{V_{base2}}{4} = \frac{480}{4} = 120$$

This choice of power and voltage bases in the three zones gives the following current and impedance bases:

$$I_{base1} = \frac{S_{base}}{V_{base1}} = \frac{30000}{240} = 125 \text{ A}$$

$$I_{base2} = \frac{S_{base}}{V_{base2}} = \frac{30000}{480} = 62.5 \text{ A}$$

$$I_{base3} = \frac{S_{base}}{V_{base3}} = \frac{30000}{120} = 250 \text{ A}$$

$$Z_{base1} = \frac{V_{base1}^2}{S_{base}} = \frac{240^2}{30000} = 1.92 \text{ } \Omega$$

$$Z_{base2} = \frac{V_{base2}^2}{S_{base}} = \frac{480^2}{30000} = 7.68 \text{ } \Omega$$

$$Z_{base3} = \frac{V_{base3}^2}{S_{base}} = \frac{120^2}{30000} = 0.48 \text{ } \Omega$$

The circuit quantities are then transformed into per-unit values.

$$V_{s,\text{pu}} = \frac{V_s}{V_{base1}} = \frac{220 \angle 0^\circ}{240} = 0.9167 \angle 0^\circ \text{ pu}$$

$$X_{line,\text{pu}} = \frac{X_{line}}{Z_{base2}} = \frac{2}{7.68} = 0.2604 \text{ pu}$$

$$Z_{load,\text{pu}} = \frac{Z_{load}}{Z_{base3}} = \frac{0.9 + j0.2}{0.48} = (1.875 + j0.4167) \text{ pu}$$

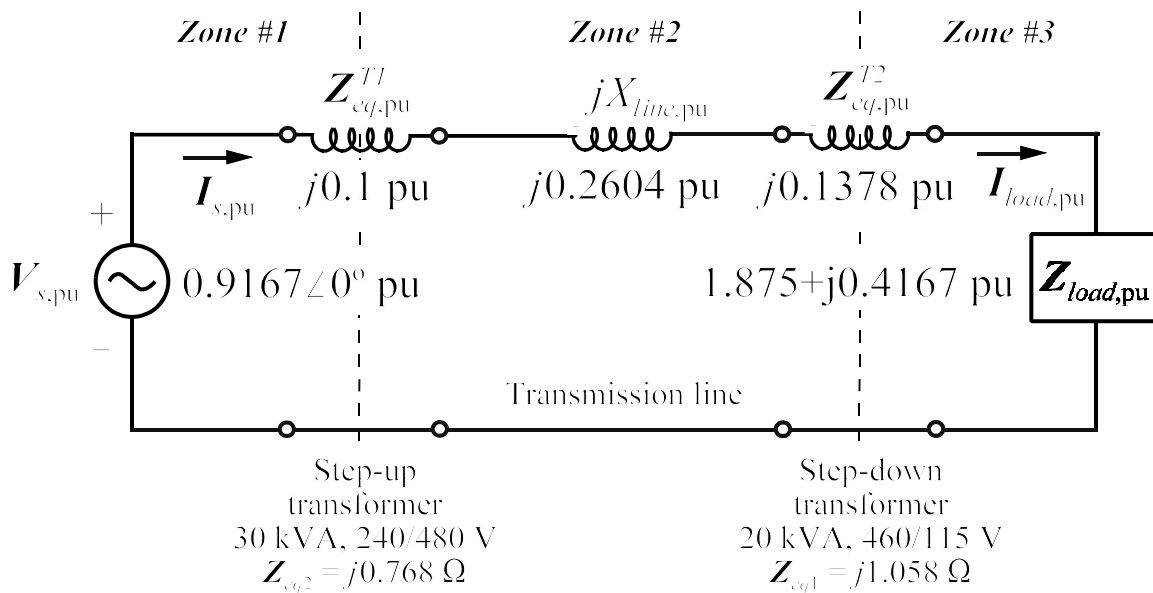
The transformer impedances are

$$\mathbf{Z}_{eq,pu}^{T1} = \frac{\mathbf{Z}_{eq2}}{\mathbf{Z}_{base2}} = \frac{j0.768}{7.68} = j0.1 \text{ pu} \quad \left(\begin{array}{l} \text{step-down} \\ \text{transformer} \end{array} \right)$$

$$\mathbf{Z}_{eq,pu}^{T2} = \frac{\mathbf{Z}_{eq1}}{\mathbf{Z}_{base2}} = \frac{j1.058}{7.68} = j0.1378 \text{ pu} \quad \left(\begin{array}{l} \text{step-up} \\ \text{transformer} \end{array} \right)$$

The overall per-unit equivalent circuit is shown below.

The per-unit load current is given by



$$\begin{aligned} \mathbf{I}_{load,pu} &= \frac{0.9167 \angle 0^\circ}{1.875 + j(0.10 + 0.2604 + 0.1378 + 0.4167)} \text{ pu} \\ &= 0.4395 \angle -26.01^\circ \text{ pu} \end{aligned}$$

The actual load current is

$$\begin{aligned} \mathbf{I}_{load} &= \mathbf{I}_{load,pu} \mathbf{I}_{base3} = (0.4395 \angle -26.01^\circ)(250 \text{ A}) \\ &= 109.9 \angle -26.01^\circ \text{ A} \end{aligned}$$