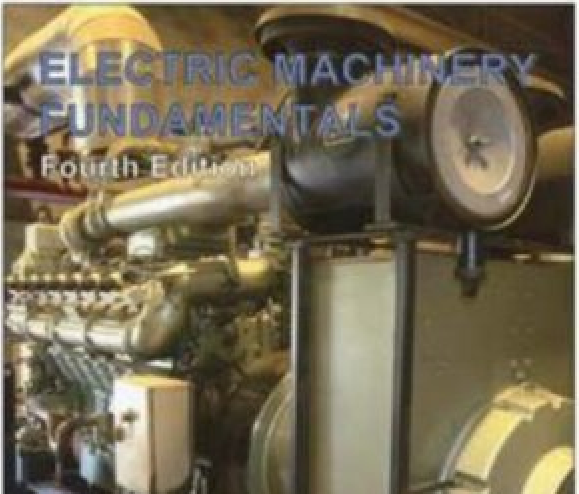
ELECTRIC MACHINERY FUNDAMENTALS

Fourth Edition



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McGRAW-HILL INTERNATIONAL EDITION

CHAPTER 1 – Introduction to Machinery Principles

Summary:

1. Basic concept of electrical machines fundamentals:
    - Rotational component measurements
        - Angular Velocity, Acceleration
        - Torque, Work, Power
        - Newton's Law of Rotation
    - Magnetic Field study
        - Production of a Magnetic Field
        - Magnetic Circuits

2. Magnetic Behaviour of Ferromagnetic Materials

3. How magnetic field can affect its surroundings:
    - Faraday's Law – Induced Voltage from a Time-Changing Magnetic Field.
    - Production of Induced Force on a Wire.
    - Induced Voltage on a Conductor moving in a Magnetic Field

4. Linear DC Machines

Introduction

1. Electric Machines → mechanical energy to electric energy or vice versa

   Mechanical energy → Electric energy : GENERATOR

   Electric energy → mechanical energy : MOTOR

2. Almost all practical motors and generators convert energy from one form to another through the action of a **magnetic field**.

3. Only machines using magnetic fields to perform such conversions will be considered in this course.

4. When we talk about machines, another related device is the transformer. A transformer is a device that converts ac electric energy at one voltage level to ac electric energy at another voltage level.

5. Transformers are usually studied together with generators and motors because they operate on the same principle, the difference is just in **the action of a magnetic field** to accomplish the change in voltage level.

6. Why are electric motors and generators so common?
   - electric power is a clean and efficient energy source that is very easy to transmit over long distances and easy to control.
   - Does not require constant ventilation and fuel (compare to internal-combustion engine), free from pollutant associated with combustion

1. Basic concept of electrical machines fundamentals

1.1 Rotational Motion, Newton's Law and Power Relationship

Almost all electric machines **rotate about an axis**, called the shaft of the machines. It is important to have a basic understanding of rotational motion.

Angular position, θ - is the angle at which it is oriented, measured from some arbitrary reference point. Its measurement units are in radians (rad) or in degrees. It is similar to the linear concept of distance along a line.

Conventional notation: +ve value for anticlockwise rotation
-ve value for clockwise rotation

Angular Velocity, ω - Defined as the velocity at which the measured point is moving. Similar to the concept of standard velocity where:

$$v = \frac{dr}{dt}$$

where:

r – distance traverse by the body
t – time taken to travel the distance r

For a rotating body, angular velocity is formulated as:

$$\omega = \frac{d\theta}{dt} \text{ (rad/s)}$$

where:

θ - Angular position/ angular distance traversed by the rotating body
t – time taken for the rotating body to traverse the specified distance, ϑ.

Angular acceleration, α - is defined as the rate of change in angular velocity with respect to time. Its formulation is as shown:

$$\alpha = \frac{d\omega}{dt} \text{ (rad/s}^2\text{)}$$

Torque, τ

1. In linear motion, a force applied to an object causes its velocity to change. In the absence of a net force on the object, its velocity is constant. The greater the force applied to the object, the more rapidly its velocity changes.

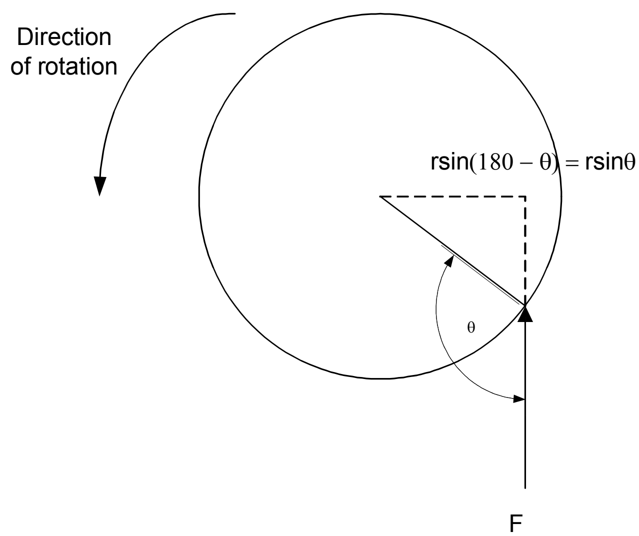
2. Similarly in the concept of rotation, when an object is rotating, its angular velocity is constant unless a torque is present on it. Greater the torque, more rapid the angular velocity changes.

3. Torque is known as a rotational force applied to a rotating body giving angular acceleration, a.k.a. 'twisting force'.

4. Definition of Torque: (Nm)

'Product of force applied to the object and the smallest distance between the line of action of the force and the object's axis of rotation'

$$\therefore \tau = \text{Force} \times \text{perpendicular distance}$$
$$= F \times r \sin\theta$$



Work, W – is defined as the application of Force through a distance. Therefore, work may be defined as:

$$W = \int F dr$$

Assuming that the direction of F is collinear (in the same direction) with the direction of motion and constant in magnitude, hence,

$$W = Fr$$

Applying the same concept for rotating bodies,

$$W = \int \tau d\theta$$

Assuming that τ is constant,

$$W = \tau\theta \text{ (Joules)}$$

Power, P – is defined as rate of doing work. Hence,

$$P = \frac{dW}{dt} \text{ (watts)}$$

Applying this for rotating bodies,

$$P = \frac{d}{dt}(\tau\theta)$$
$$= \tau\frac{d\theta}{dt}$$
$$= \tau\omega$$

This equation can describe the mechanical power on the shaft of a motor or generator.

Newton's Law of Rotation

Newton's law for objects moving in a straight line gives a relationship between the force applied to the object and the acceleration experience by the object as the result of force applied to it. In general,

$$F = ma$$

where:
- F – Force applied
- m – mass of object
- a – resultant acceleration of object

Applying these concept for rotating bodies,

$$\tau = J\alpha \text{ (Nm)}$$

where:
- τ - Torque
- J – moment of inertia
- α - angular acceleration

1.2 The Magnetic Field

Magnetic fields are the **fundamental mechanism by which energy is converted** from one form to another in motors, generators and transformers.

First, we are going to look at the basic principle – **A current-carrying wire produces a magnetic field in the area around it.**

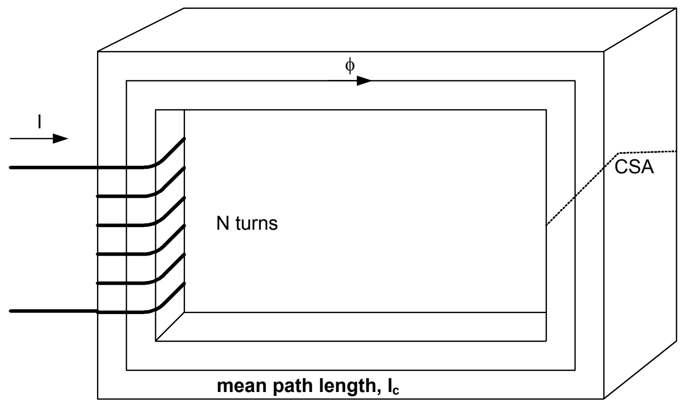
Production of a Magnetic Field

1. **Ampere's Law –** the basic law governing the production of a magnetic field by a current:

$$\oint H\,dl = I_{net}$$

where **H** is the magnetic field intensity produced by the current I_{net} and *dl* is a differential element of length along the path of integration. H is measured in Ampere-turns per meter.

2. Consider a current currying conductor is wrapped around a ferromagnetic core;



3. Applying Ampere's law, the total amount of magnetic field induced will be proportional to the amount of current flowing through the conductor wound with N turns around the ferromagnetic material as shown. Since the core is made of ferromagnetic material, it is assume that a majority of the magnetic field will be confined to the core.

4. The path of integration in Ampere's law is the mean path length of the core, l_c. The current passing within the path of integration I_{net} is then Ni, since the coil of wires cuts the path of integration N times while carrying the current i. Hence Ampere's Law becomes,

$$Hl_c = Ni$$

$$\therefore H = \frac{Ni}{l_c}$$

5. In this sense, H (Ampere turns per metre) is known as the effort required to induce a magnetic field. The strength of the magnetic field flux produced in the core also depends on the material of the core. Thus,

$$B = \mu H$$

B = magnetic flux density (webers per square meter, Tesla (T))
μ= magnetic permeability of material (Henrys per meter)
H = magnetic field intensity (ampere-turns per meter)

6. The constant μ may be further expanded to include *relative permeability* which can be defined as below:

$$\mu_r = \frac{\mu}{\mu_o}$$

where: μ_o – permeability of free space (a.k.a. air)

7. Hence the permeability value is a combination of the relative permeability and the permeability of free space. The value of relative permeability is dependent upon the type of material used. The higher the amount permeability, the higher the amount of flux induced in the core. Relative permeability is a convenient way to compare the magnetizability of materials.

8. Also, because the permeability of iron is so much higher than that of air, the majority of the flux in an iron core remains inside the core instead of travelling through the surrounding air, which has lower permeability. The small leakage flux that does leave the iron core is important in determining the flux linkages between coils and the self-inductances of coils in transformers and motors.

9. In a core such as in the figure,

$$B = \mu H = \frac{\mu N i}{l_c}$$

Now, to measure the total flux flowing in the ferromagnetic core, consideration has to be made in terms of its cross sectional area (CSA). Therefore,

$$\phi = \int_A B dA$$

Where: A – cross sectional area throughout the core

Assuming that the flux density in the ferromagnetic core is constant throughout hence constant A, the equation simplifies to be:

$$\phi = BA$$

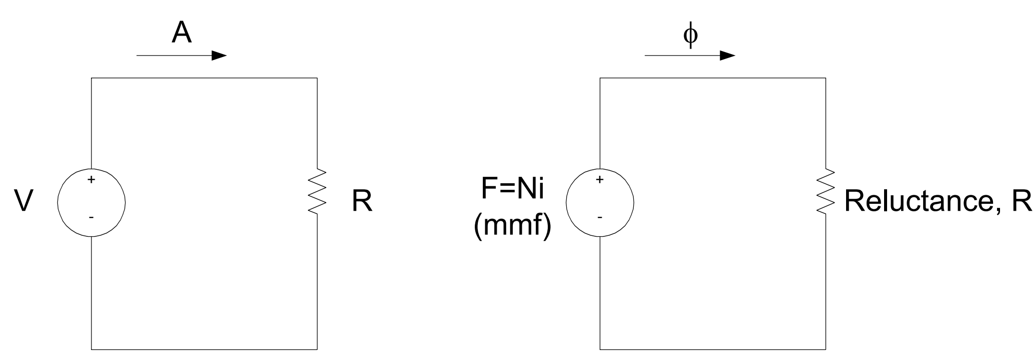
Taking into account past derivation of B,

$$\phi = \frac{\mu N i A}{l_c}$$

2. <u>Magnetics Circuits</u>

The flow of magnetic flux induced in the ferromagnetic core can be made analogous to an electrical circuit hence the name magnetic circuit.

The analogy is as follows:



Electric Circuit Analogy *Magnetic Circuit Analogy*

1. Referring to the magnetic circuit analogy, F is denoted as **magnetomotive force** (mmf) which is similar to Electromotive force in an electrical circuit (emf). Therefore, we can safely say that F is the prime mover or force which pushes magnetic flux around a ferromagnetic core at a value of Ni (refer to ampere's law). Hence F is measured in ampere turns. Hence the magnetic circuit equivalent equation is as shown:

$$F = \phi R$$ (similar to V=IR)

2. The polarity of the mmf will determine the direction of flux. To easily determine the direction of flux, the 'right hand curl' rule is utilised:

a) The direction of the curled fingers determines the current flow.
b) The resulting thumb direction will show the magnetic flux flow.

3. The element of R in the magnetic circuit analogy is similar in concept to the electrical resistance. It is basically the measure of material resistance to the flow of magnetic flux. **Reluctance** in this analogy obeys the rule of electrical resistance (Series and Parallel Rules). Reluctance is measured in Ampere-turns per weber.

Series Reluctance,

$$Req = R1 + R2 + R3 + \dots$$

Parallel Reluctance,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

4. The inverse of electrical resistance is conductance which is a measure of conductivity of a material. Hence the inverse of reluctance is known as **permeance, P** where it represents the degree at which the material permits the flow of magnetic flux.

$$P = \frac{1}{R}$$

$$\therefore \text{since } \phi = \frac{F}{R}$$

$$\therefore \phi = FP$$

Also,

$$\phi = \frac{\mu NiA}{l_c}$$

$$= Ni\frac{\mu A}{l_c}$$

$$= F\frac{\mu A}{l_c}$$

$$\therefore P = \frac{\mu A}{l_c}, \; R = \frac{l_c}{\mu A}$$

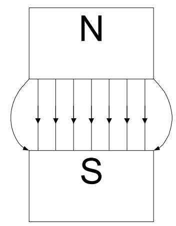
5. By using the magnetic circuit approach, it simplifies calculations related to the magnetic field in a ferromagnetic material, however, this approach has inaccuracy embedded into it due to assumptions made in creating this approach (within 5% of the real answer). Possible reason of inaccuracy is due to:

a) The magnetic circuit assumes that all flux are confined within the core, but in reality a small fraction of the flux escapes from the core into the surrounding low-permeability air, and this flux is called **leakage flux**.

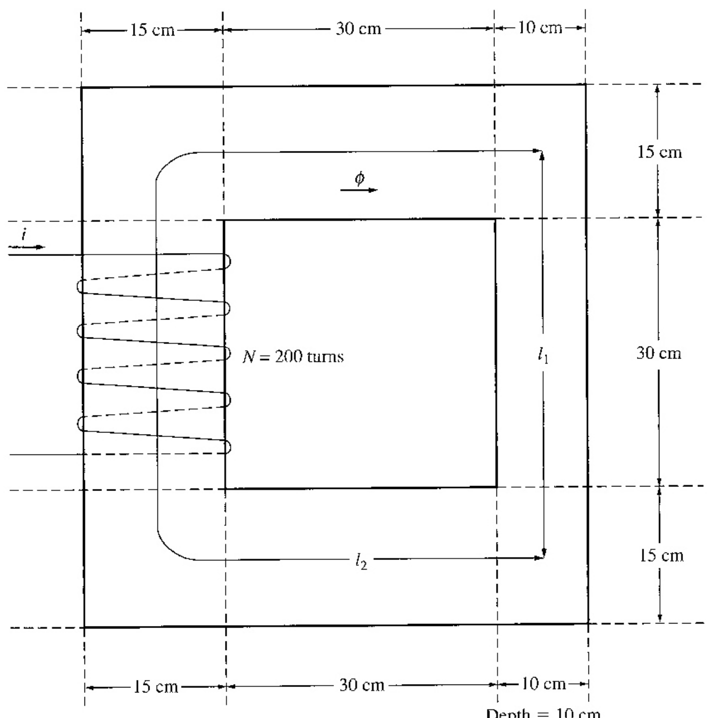
b) The reluctance calculation assumes a certain mean path length and cross sectional area (csa) of the core. This is alright if the core is just one block of ferromagnetic material with no corners, for practical ferromagnetic cores which have **corners** due to its design, this assumption is not accurate.

c) In ferromagnetic materials, the permeability varies with the amount of flux already in the material. The material permeability is not constant hence there is an existence of **non-linearity of permeability.**

d) For ferromagnetic core which has air gaps, there are **fringing effects** that should be taken into account as shown:



<u>Example 1.1</u>

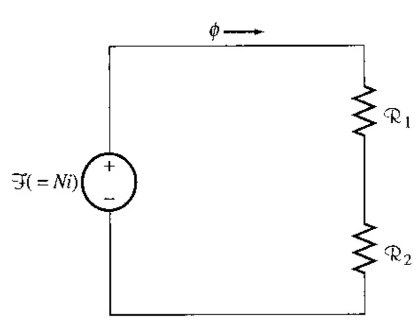


(a)

A ferromagnetic core is shown. Three sides of this core are of uniform width, while the fourth side is somewhat thinner. The depth of the core (into the page) is 10cm, and the other dimensions are shown in the figure. There is a 200 turn coil wrapped around the left side of the core. Assuming relative permeability μ_r of 2500, **how much flux will be produced** by a 1A input current?

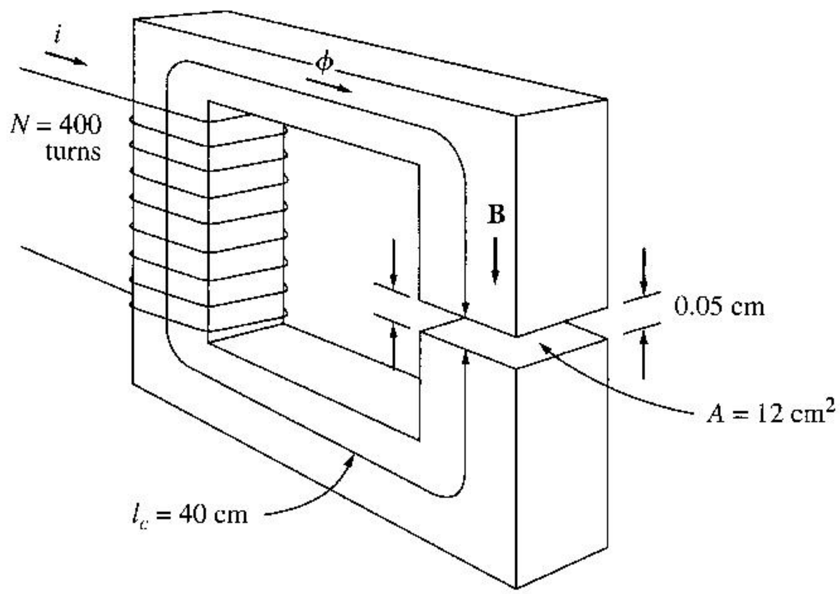
Solution:
3 sides of the core have the same csa, while the 4th side has a different area. Thus the core can be divided into 2 regions:
(1) the single thinner side
(2) the other 3 sides taken together

The magnetic circuit corresponding to this core:



(b)

Example 1.2



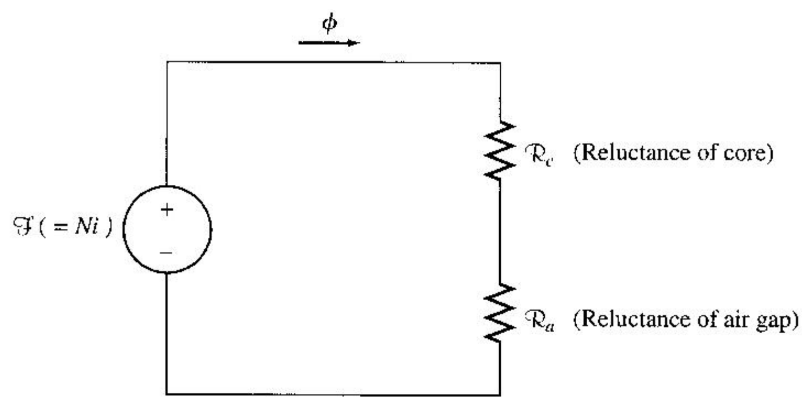
(a)

Figure shows a ferromagnetic core whose mean path length is 40cm. There is a small gap of 0.05cm in the structure of the otherwise whole core. The csa of the core is 12cm^2, the relative permeability of the core is 4000, and the coil of wire on the core has 400 turns. Assume that fringing in the air gap increases the effective csa of the gap by 5%. Given this information, find

(a) the **total reluctance** of the flux path (iron plus air gap)
(b) the **current** required to produce a flux density of 0.5T in the air gap.

Solution:

The magnetic circuit corresponding to this core is shown below:



(b)

Example 1.3

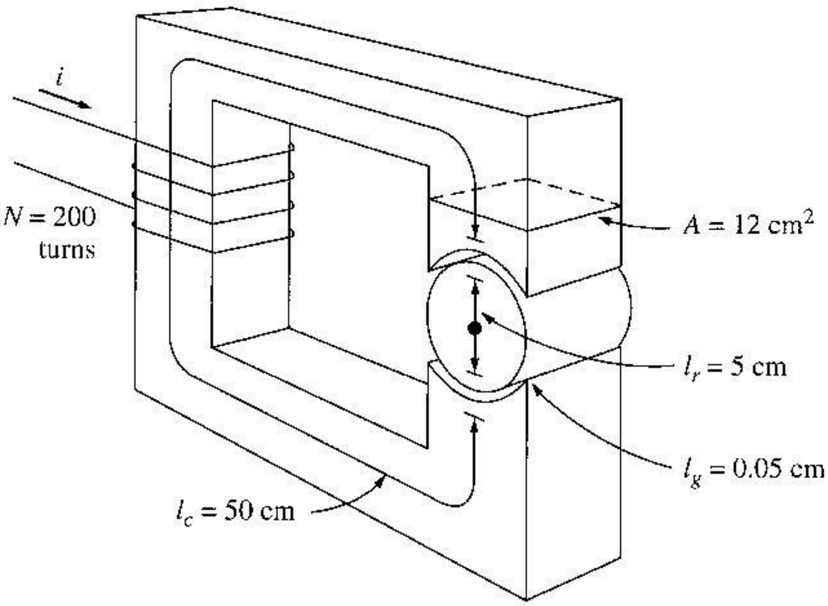
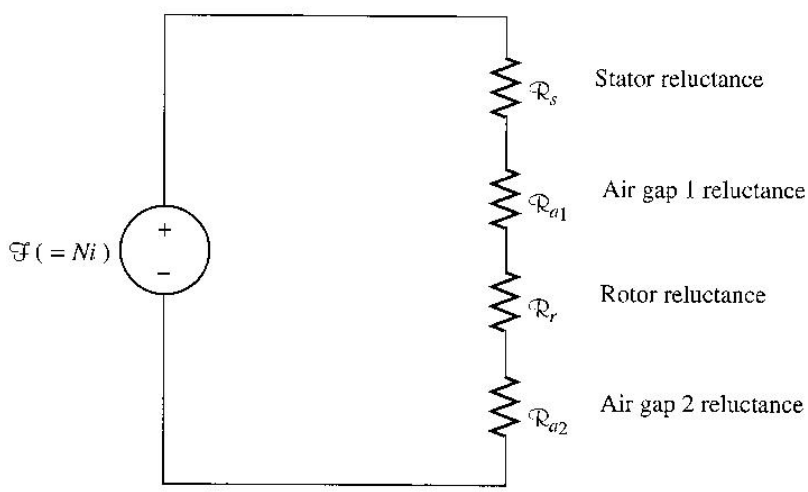


Figure shows a simplified rotor and stator for a dc motor. The mean path length of the stator is 50cm, and its csa is $12cm^2$. The mean path length of the rotor is 5 cm, and its csa also may be assumed to be $12cm^2$. Each air gap between the rotor and the stator is 0.05cm wide, and the csa of each air gap (including fringing) is $14cm^2$. The iron of the core has a relative permeability of 2000, and there are 200 turns of wire on the core. If the current in the wire is adjusted to be 1A, what will the **resulting flux density in the air gaps** be?

Solution:

To determine the flux density in the air gap, it is necessary to first calculate the mmf applied to the core and the total reluctance of the flux path. With this information, the total flux in the core can be found. Finally, knowing the csa of the air gaps enables the flux density to be calculated.

The magnetic cct corresponding to this machine is shown below.



(b)

Magnetic Behaviour of Ferromagnetic Materials

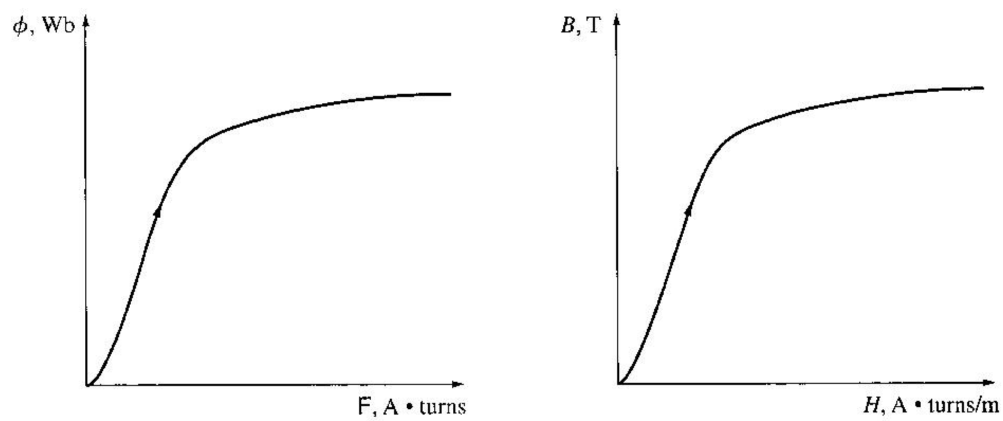
1. Materials which are classified as non-magnetic all show a linear relationship between the flux density B and coil current I. In other words, they have constant permeability. Thus, for example, in free space, the permeability is constant. But in iron and other ferromagnetic materials it is not constant.

2. For magnetic materials, a much larger value of B is produced in these materials than in free space. Therefore, the permeability of magnetic materials is much higher than μ_o. However, the permeability is not linear anymore but does depend on the current over a wide range.

3. Thus, the **permeability is the property of a medium that determines its magnetic characteristics**. In other words, the concept of magnetic permeability corresponds to the ability of the material to permit the flow of magnetic flux through it.

4. In electrical machines and electromechanical devices a somewhat linear relationship between B and I is desired, which is normally approached by limiting the current.

5. Look at the magnetization curve and B-H curve. Note: The curve corresponds to an increase of DC current flow through a coil wrapped around the ferromagnetic core (ref: Electrical Machinery Fundamentals 4th Ed. – Stephen J Chapman).



(a) (b)

6. When the flux produced in the core is plotted versus the mmf producing it, the resulting plot looks like this (a). This plot is called a **saturation curve** or a **magnetization curve.** A small increase in the mmf produces a huge increase in the resulting flux. After a certain point, further increases in the mmf produce relatively smaller increases in the flux. Finally, there will be no change at all as you increase mmf further. The region in which the curve flattens out is called saturation region, and the core is said to be saturated. The region where the flux changes rapidly is called **the unsaturated region**. The transition region is called the 'knee' of the curve.

7. From equation $H = Ni/l_c = F/l_c$ and ϕ =BA, it can be seen that magnetizing intensity is directly proportional to mmf and magnetic flux density is directly proportional to flux for any given core. $B=\mu H$ → slope of curve is the permeability of the core at that magnetizing intensity. The curve (b) shows that the permeability is large and relatively constant in the unsaturated region and then gradually drops to a low value as the core become heavily saturated.

8. Advantage of using a ferromagnetic material for cores in electric machines and transformers is that one gets more flux for a given mmf than with air (free space).

9. If the resulting flux has to be proportional to the mmf, then the core must be operated in the unsaturated region.

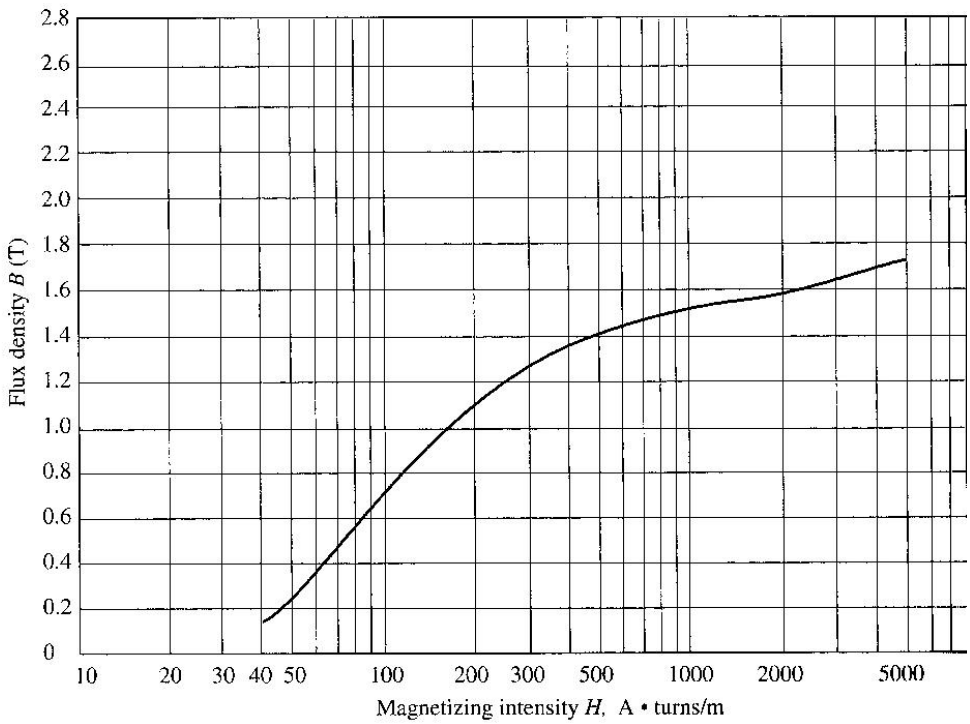
10. Generators and motors depend on magnetic flux to produce voltage and torque, so they need as much flux as possible. So, they operate near the knee of the magnetization curve (flux not linearly related to the mmf). This non-linearity as a result gives peculiar behaviours to machines.

11. As magnetizing intensity H increased, the relative permeability first increases and then starts to drop off.

Example 1.5

A square magnetic core has a mean path length of 55cm and a csa of $150cm^2$. A 200 turn coil of wire is wrapped around one leg of the core. The core is made of a material having the magnetization curve shown below. Find:

a) How much current is required to produce 0.012 Wb of flux in the core?
b) What is the core's relative permeability at that current level?
c) What is its reluctance?

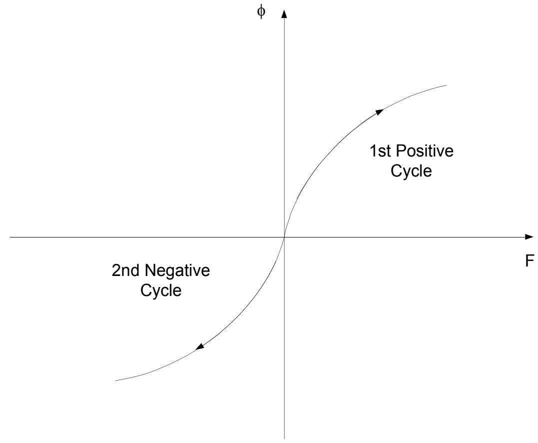


(c)

Energy Losses in a Ferromagnetic Core

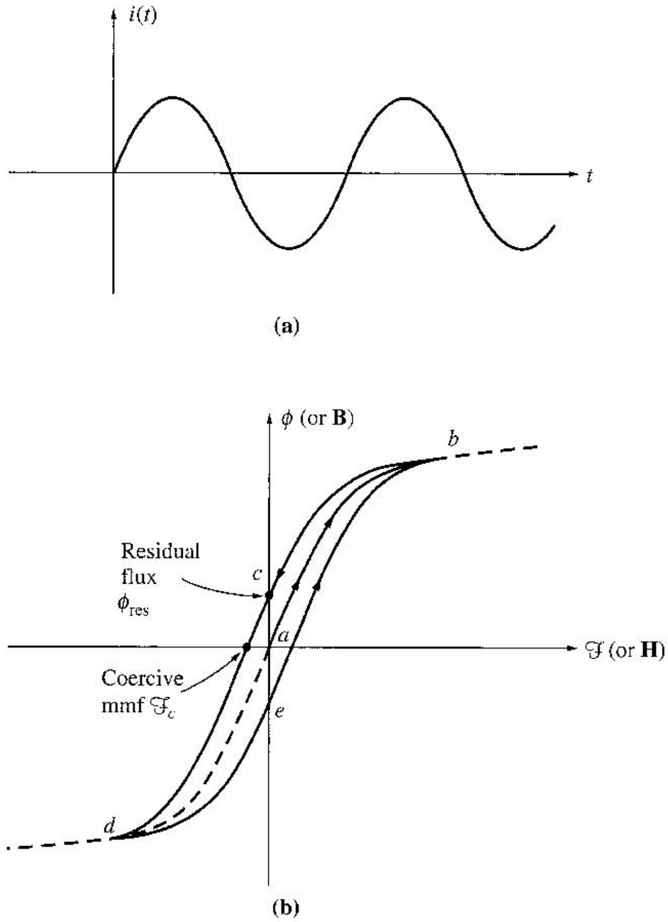
I. Hysteresis Loss

1. Discussions made before concentrates on the application of a DC current through the coil. Now let's move the discussion into the application of AC current source at the coil. Using our understanding previously, we can predict that the curve would be as shown,



Theoretical ac magnetic behaviour for flux in a ferromagnetic core.

2. Unfortunately, the above assumption is only correct provided that the core is 'perfect' i.e. there are no residual flux present during the negative cycle of the ac current flow. A typical flux behaviour (or known as hysteresis loop) in a ferromagnetic core is as shown in the next page.



Typical Hysterisis loop when ac current is applied.

3. <u>Explanation of Hysteresis Loop</u>

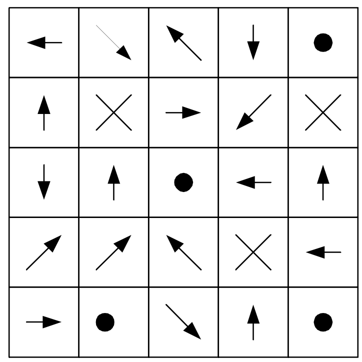
- Apply AC current. Assume flux in the core is initially zero.
- As current increases, the flux traces the path *ab.* (saturation curve)
- When the current decreases, the flux traces out a different path from the one when the current increases.
- When current decreases, the flux traces out path *bcd*.
- When the current increases again, it traces out path *deb*.
- NOTE: the amount of flux present in the core depends not only on the amount of current applied to the windings of the core, but also on the previous history of the flux in the core.
- HYSTERESIS is the dependence on the preceding flux history and the resulting failure to retrace flux paths.
- When a large mmf is first applied to the core and then removed, the flux path in the core will be *abc*.
- When mmf is removed, the flux does not go to zero – **residual flux**. This is how permanent magnets are produced.
- To force the flux to zero, an amount of mmf known as **coercive mmf** must be applied in the opposite direction.

4. <u>Why does hysteresis occur?</u>

- To understand hysteresis in a ferromagnetic core, we have to look into the behaviour of its atomic structure before, during and after the presence of a magnetic field.

- The atoms of iron and similar metals (cobalt, nickel, and some of their alloys) tend to have their magnetic fields closely aligned with each other. Within the metal, there is an existence of small regions known as **domains** where in each domain there is a presence of a small magnetic field which randomly aligned through the metal structure.

This as shown below:



An example of a magnetic domain orientation in a metal structure before the presence of a magnetic field.

- Magnetic field direction in each domain is random as such that the net magnetic field is zero.

- When mmf is applied to the core, each magnetic field will align with respect to the direction of the magnetic field. That explains the exponential increase of magnetic flux during the early stage of magnetisation. As more and more domain are aligned to the magnetic field, the total magnetic flux will maintain at a constant level hence as shown in the magnetisation curve (saturation).

- When mmf is removed, the magnetic field in each domain **will try** to revert to its random state.

- However, **not all** magnetic field domain's would revert to its random state hence it remained in its previous magnetic field position. This is due to the lack of energy required to disturb the magnetic field alignment.

- Hence the material will retain some of its magnetic properties (permanent magnet) up until an external energy is applied to the material. Examples of external energy may be in the form of heat or large mechanical shock. That is why a permanent magnet can lose its magnetism if it is dropped, hit with a hammer or heated.

- Therefore, in an ac current situation, to realign the magnetic field in each domain during the opposite cycle would require extra mmf (also known as coercive mmf).

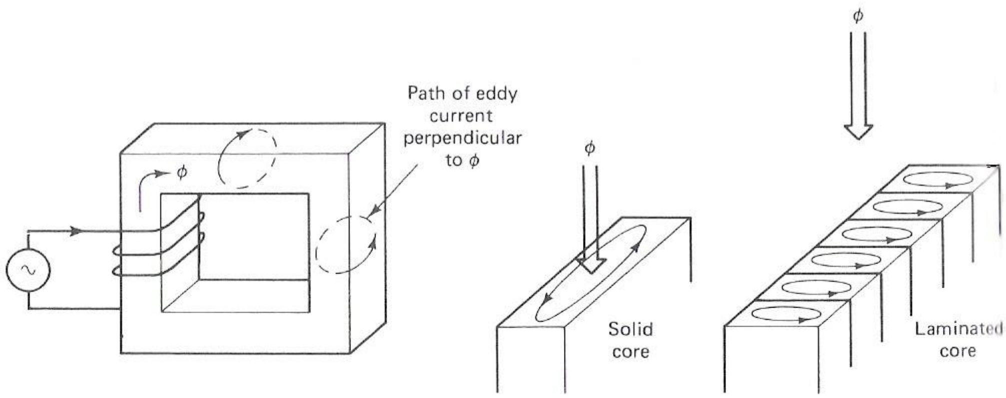
- This extra energy requirement is known as **hysteresis loss**.

- The larger the material, the more energy is required hence the higher the hysteresis loss.

- Area enclosed in the hysteresis loop formed by applying an ac current to the core is directly proportional to the energy lost in a given ac cycle.

II. <u>Eddy Current Loss</u>

1. A time-changing flux induces voltage within a ferromagnetic core.
2. These voltages cause swirls of current to flow within the core – eddy currents.
3. Energy is dissipated (in the form of heat) because these eddy currents are flowing in a resistive material (iron)
4. The amount of energy lost to eddy currents is proportional to the **size of the paths** they follow within the core.
5. To reduce energy loss, ferromagnetic core should be broken up into small strips, or laminations, and build the core up out of these strips. An insulating oxide or resin is used between the strips, so that the current paths for eddy currents are limited to small areas.



Conclusion:

Core loss is extremely important in practice, since it greatly affects operating temperatures, efficiencies, and ratings of magnetic devices.

3. <u>How Magnetic Field can affect its surroundings</u>

3.1 <u>FARADAY'S LAW – Induced Voltage from a Time-Changing Magnetic Field</u>

Before, we looked at the production of a magnetic field and on its properties. Now, we will look at the various ways in which an existing magnetic field can affect its surroundings.

1. Faraday's Law:

'If a flux passes through a turn of a coil of wire, voltage will be induced in the turn of the wire that is directly proportional to the rate of change in the flux with respect of time'

$$e_{ind} = -\frac{d\phi}{dt}$$

If there is N number of turns in the coil with the same amount of flux flowing through it, hence:

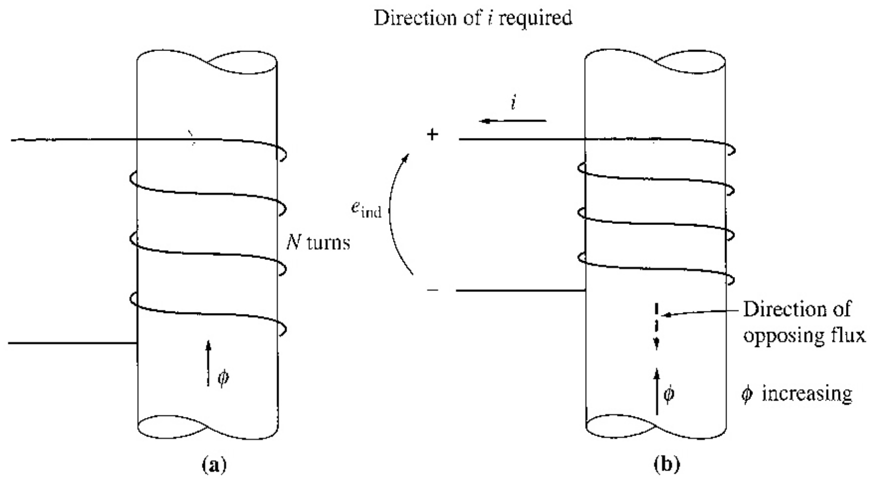
$$e_{ind} = -N\frac{d\phi}{dt}$$

where: N – number of turns of wire in coil.

Note the negative sign at the equation above which is in accordance to **Lenz' Law** which states:

'The direction of the build-up voltage in the coil is as such that if the coils were short circuited, it would produce current that would cause a flux opposing the original flux change.'

Examine the figure below:



- If the ~~flux shown~~ is increasing in strength, then the voltage built up in the coil will **tend to establish a flux that will oppose the increase.**
- A current flowing as shown in the figure would produce a flux opposing the increase.
- So, the voltage on the coil must be built up with the polarity required to drive the current through the external circuit. So, $-e_{ind}$
- NOTE: In Chapman, the minus sign is often left out because the polarity of the resulting voltage can be determined from physical considerations.

2. Equation $e_{ind} = -d\phi/dt$ assumes that exactly the same flux is present in each turn of the coil. This is not true, since there is leakage flux. This equation will give valid answer if the windings are tightly coupled, so that the vast majority of the flux passing thru one turn of the coil does indeed pass through all of them.

3. Now consider the induced voltage in the *i*th turn of the coil,

$$e_i = \frac{d\phi_i}{dt}$$

Since there is N number of turns,

$$e_{ind} = \sum_{i=1}^{N} e_i$$

$$= \sum_{i=1}^{N} \frac{d\phi_i}{dt}$$

$$= \frac{d}{dt}\left(\sum_{i=1}^{N} \phi_i\right)$$

The equation above may be rewritten into,

$$e_{ind} = \frac{d\lambda}{dt}$$

where λ (flux linkage) is defined as:

$$\lambda = \sum_{i=1}^{N} \phi_i \quad \text{(weber-turns)}$$

4. Faraday's law is the fundamental property of magnetic fields involved in transformer operation.

5. Lenz's Law in transformers is used to predict the polarity of the voltages induced in transformer windings.

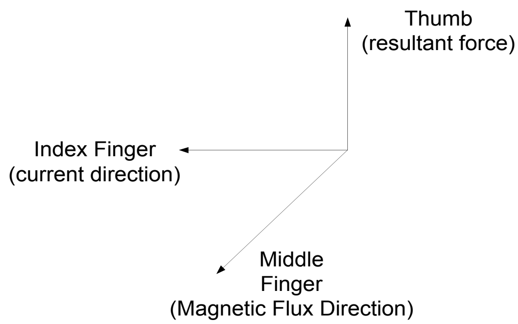
3.2 Production of Induced Force on a Wire.

1. A current carrying conductor present in a uniform magnetic field of flux density B, would produce a force to the conductor/wire. Dependent upon the direction of the surrounding magnetic field, the force induced is given by:

$$F = i\left(l \times B\right)$$

where:
i – represents the current flow in the conductor
l – length of wire, with direction of *l* defined to be in the direction of current flow
B – magnetic field density

2. The direction of the force is given by the right-hand rule. Direction of the force depends on the direction of current flow and the direction of the surrounding magnetic field. A rule of thumb to determine the direction can be found using the right-hand rule as shown below:



Right Hand rule

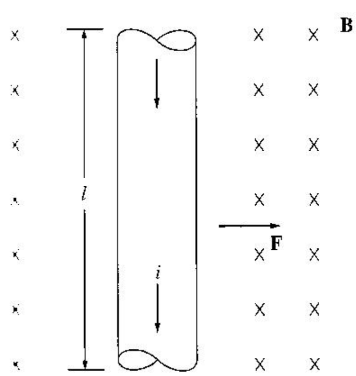
3. The induced force formula shown earlier is true if the current carrying conductor is perpendicular to the direction of the magnetic field. If the current carrying conductor is position at an angle to the magnetic field, the formula is modified to be as follows:

$$F = ilB\sin\theta$$

Where: θ - angle between the conductor and the direction of the magnetic field.

4. In summary, this phenomenon is the basis of an **electric motor** where torque or rotational force of the motor is the effect of the stator field current and the magnetic field of the rotor.

Example 1.7



The figure shows a wire carrying a current in the presence of a magnetic field. The magnetic flux density is 0.25T, directed into the page. If the wire is 1m long and carries 0.5A of current in the direction from the top of the page to the bottom, what are the magnitude and direction of the force induced on the wire?

3.3 Induced Voltage on a Conductor Moving in a Magnetic Field

1. If a conductor moves or 'cuts' through a magnetic field, voltage will be induced between the terminals of the conductor at which the magnitude of the induced voltage is dependent upon the velocity of the wire assuming that the magnetic field is constant. This can be summarised in terms of formulation as shown:

$$e_{ind} = (v \times B)\, l$$

   where:
   v – velocity of the wire
   B – magnetic field density
   l – length of the wire in the magnetic field

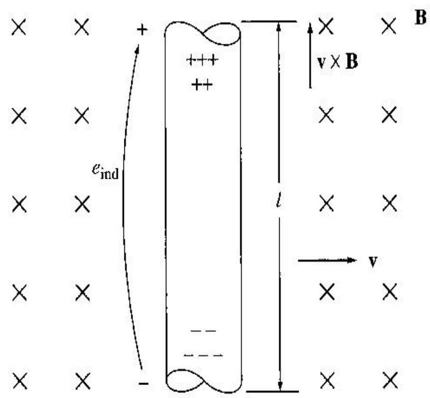
2. Note: The value of l (length) is dependent upon the angle at which the wire cuts through the magnetic field. Hence a more complete formula will be as follows:

$$e_{ind} = (v \times B) l \cos\theta$$

   where:
   θ - angle between the conductor and the direction of (***v x B***)

3. The induction of voltages in a wire moving in a magnetic field is fundamental to the operation of all types of **generators**.

Example 1.8



The figure shows a conductor moving with a velocity of 5m/s to the right in the presence of a magnetic field. The flux density is 0.5T into the page, and the wire is 1m length, oriented as shown. What are the magnitude and polarity of the resulting induced voltage?

Example 1.9

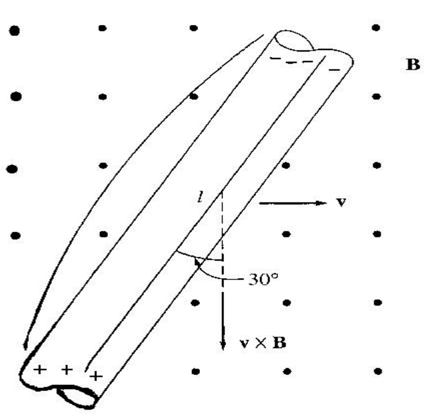
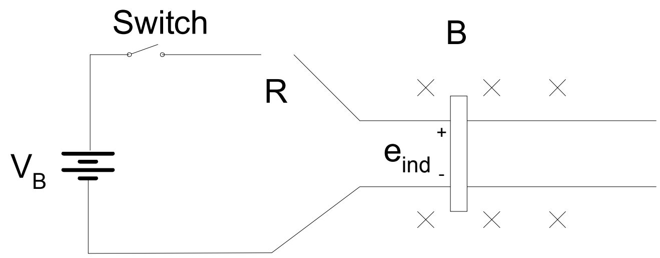


Figure shows a conductor moving with a velocity of 10m/s to the right in a magnetic field. The flux density is 0.5T, out of the page, and the wire is 1m in length. What are the magnitude and polarity of the resulting induced voltage?

4. The Linear DC Machine

Linear DC machine is the simplest form of DC machine which is easy to understand and it operates according to the same principles and exhibits the same behaviour as motors and generators. Consider the following:



Equations needed to understand linear DC machines are as follows:

Production of Force on a current carrying conductor

$$F = i(l \times B)$$

Voltage induced on a current carrying conductor moving in a magnetic field

$$e_{ind} = (v \times B)\, l$$

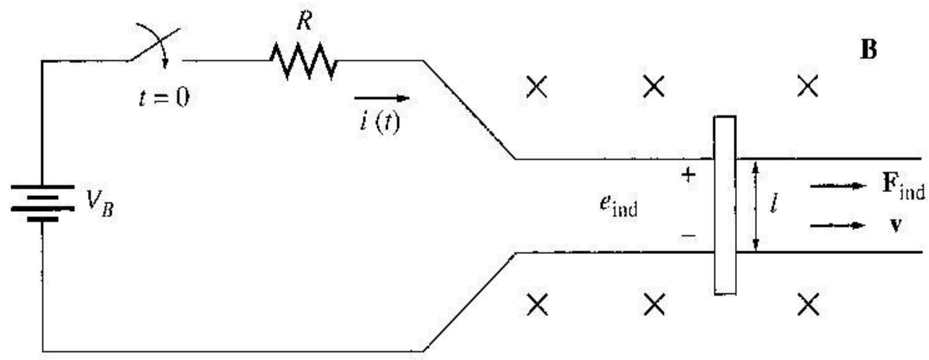
Kirchoff's voltage law

$$V_B - iR - e_{ind} = 0$$
$$\therefore V_B = e_{ind} + iR = 0$$

Newton's Law for motion

$$F_{net} = ma$$

Starting the Linear DC Machine



1. To start the machine, the switch is closed.

2. Current will flow in the circuit and the equation can be derived from Kirchoff's law:

$$Since, V_B = iR + e_{ind}$$
$$\therefore \quad i = \frac{V_B - e_{ind}}{R}$$

At this moment, the induced voltage is 0 due to no movement of the wire (the bar is at rest).

3. As the current flows down through the bar, a force will be induced on the bar. (Section 1.6 a current flowing through a wire in the presence of a magnetic field induces a force in the wire).

$$\begin{aligned} F &= i\,(l \times B) \\ &= ilB \sin 90 \\ &= ilB \end{aligned}$$

Direction of movement: Right

4. When the bar starts to move, its velocity will increase, and a voltage appears across the bar.

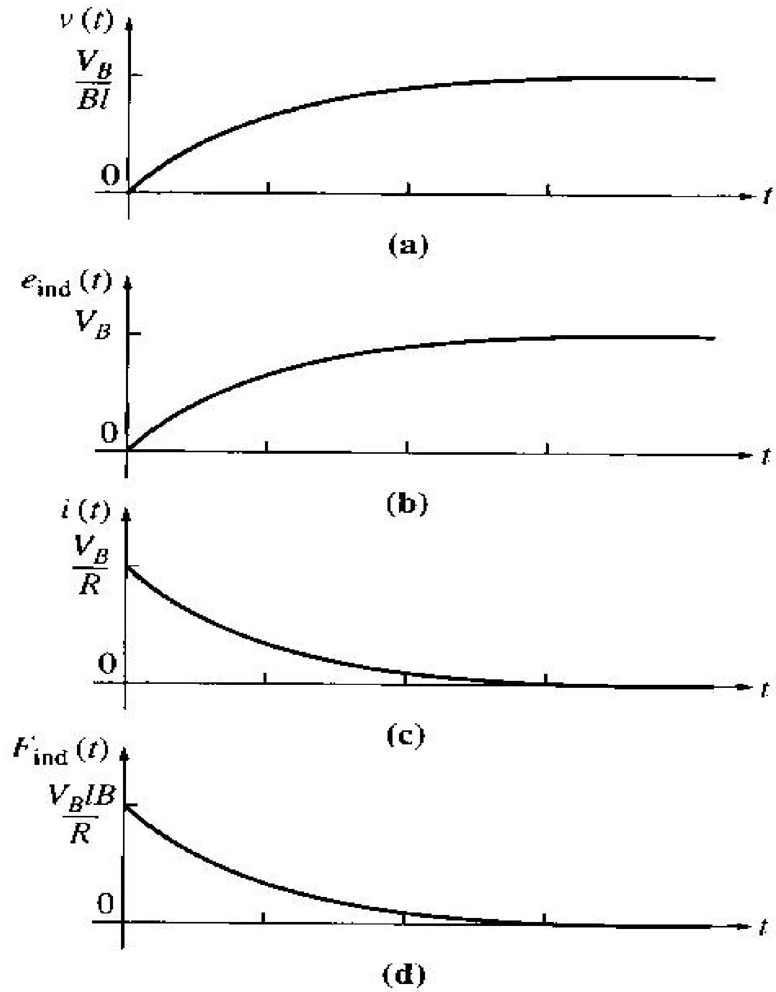
$$\begin{aligned} e_{ind} &= (v \times B)l \\ &= vBl \sin 90° \\ &= ilB \end{aligned}$$

Direction of induced potential: positive upwards

5. Due to the presence of motion and induced potential (e_{ind}), the current flowing in the bar will reduce (according to Kirchhoff's voltage law). The result of this action is that eventually the bar will reach a constant steady-state speed where the net force on the bar is zero. This occurs when e_{ind} has risen all the way up to equal V_B. This is given by:

$$V_B = e_{ind} = v_{steady\ state} Bl$$
$$\therefore v_{steady\ state} = \frac{V_B}{Bl}$$

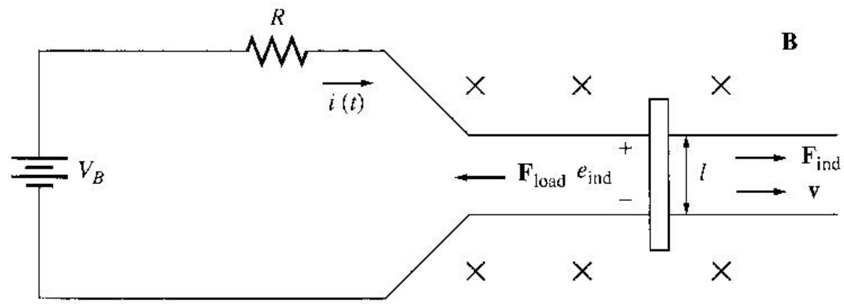
6. The above equation is true assuming that R is very small. The bar will continue to move along at this no-load speed forever unless some external force disturbs it. Summarization of the starting of linear DC machine is sketched in the figure below:



The Linear DC Machine as a Motor

1. Assume the linear machine is initially running at the no-load steady state condition (as before).

2. What happen when an external load is applied? See figure below:



3. A force F_{load} is applied to the bar opposing the direction of motion. Since the bar was initially at steady state, application of the force F_{load} will result in a net force on the bar in the direction opposite the direction of motion.

$$F_{net} = F_{load} - F_{ind}$$

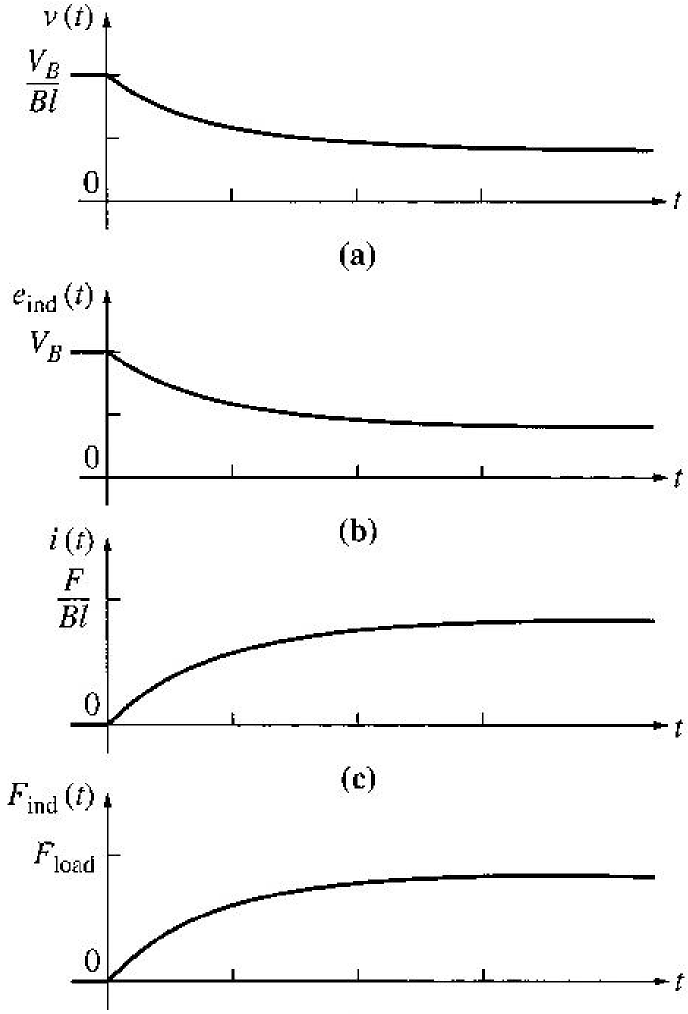
4. Thus, the bar will slow down (the resulting acceleration $a = F_{net}/m$ is negative). As soon as that happen, the induced voltage on the bar drops ($e_{ind} = v\downarrow Bl$).

5. When the induced voltage drops, the current flow in the bar will rise:

$$i\uparrow = \frac{V_B - e_{ind}\downarrow}{R}$$

6. Thus, the induced force will rise too. ($F_{ind}\uparrow = i\uparrow lB$)

7. Final result → the induced force will rise until it is equal and opposite to the load force, and the bar again travels in steady state condition, but at a lower speed. See graphs below:



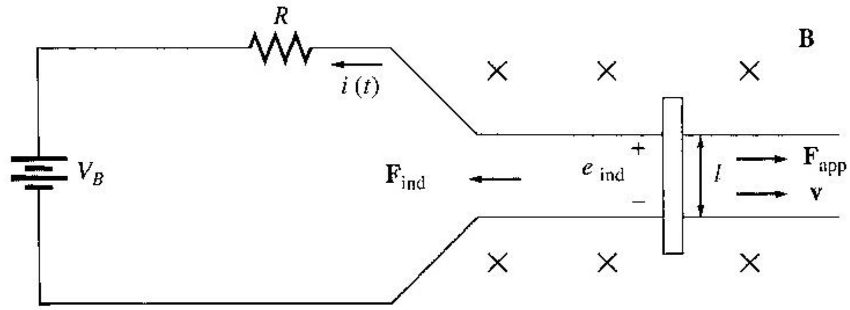
8. Now, there is an induced force in the direction of motion and power is being converted from **electrical to mechanical form** to keep the bar moving.

9. The power converted is $P_{conv} = e_{ind} I = F_{ind} v$ → An amount of electric power equal to $e_{ind}\, i$ is consumed and is replaced by the mechanical power $F_{ind}\, v$ → **MOTOR**

10. The power converted in a real rotating motor is: $P_{conv} = \tau_{ind}\, \omega$

The Linear DC Machine as a Generator

1. Assume the linear machine is operating under no-load steady-state condition. A force in the direction of motion is applied.



2. The applied force will cause the bar to accelerate in the direction of motion, and the velocity *v* will increase.

3. When the velocity increase, $e_{ind} = V \uparrow Bl$ will increase and will be larger than V_B.

4. When $e_{ind} > V_B$ the current reverses direction.

5. Since the current now flows up through the bar, it induces a force in the bar ($F_{ind} = ilB$ to the left). This induced force opposes the applied force on the bar.

6. End result → the induced force will be equal and opposite to the applied force, and the bar will move at a higher speed than before. The linear machine no is converting mechanical power $F_{ind}\ v$ to electrical power $e_{ind}\ i$ → GENERATOR

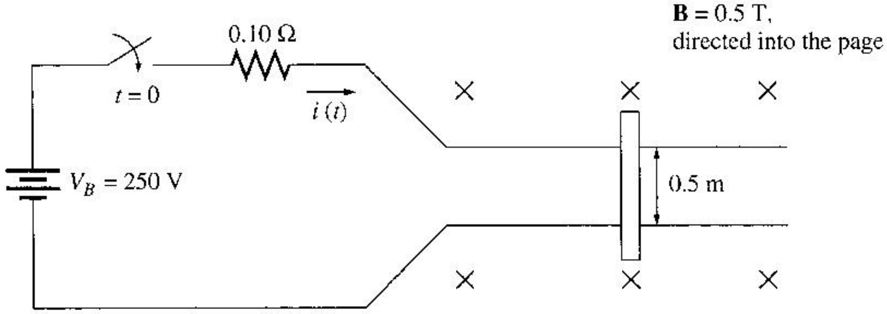
7. The amount of power converted : $P_{conv} = \tau_{ind}\ \omega$

NOTE:

- The same machine acts as both motor and generator. The only difference is whether the externally applied force is in the direction of motion (generator) or opposite to the direction of motion (motor).
- Electrically, $e_{ind} > V_B$ → generator
- $e_{ind} < V_B$ → motor
- whether the machine is a motor or a generator, both induced force (motor action) or induced voltage (generator action) is present at all times.
- Both actions are present, and it is only the **relative directions** of the **external forces with respect to the direction of motion** that determine whether the overall machine behaves as a motor or as a generator.
- The machine was a generator when it moved rapidly and a motor when it moved more slowly. But, whether it was a motor or a generator, it always moved in the same direction.
- There is a merely a small change in operating speed and a reversal of current flow.

Starting problems with the Linear Machine

1. Look at the figure here:



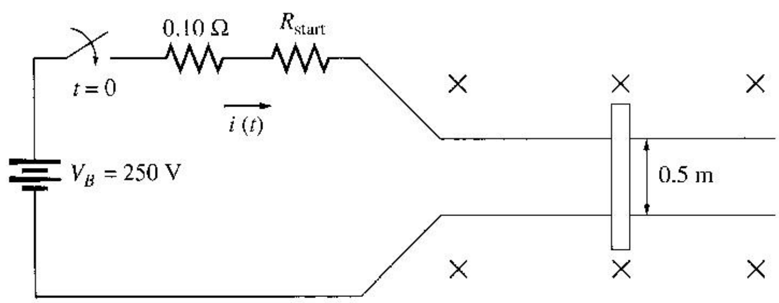
2. This machine is supplied by a 250V dc source and internal resistance R is 0.1 ohm.

3. At starting, the speed of the bar is zero, $e_{ind} = 0$. The current flow at start is:

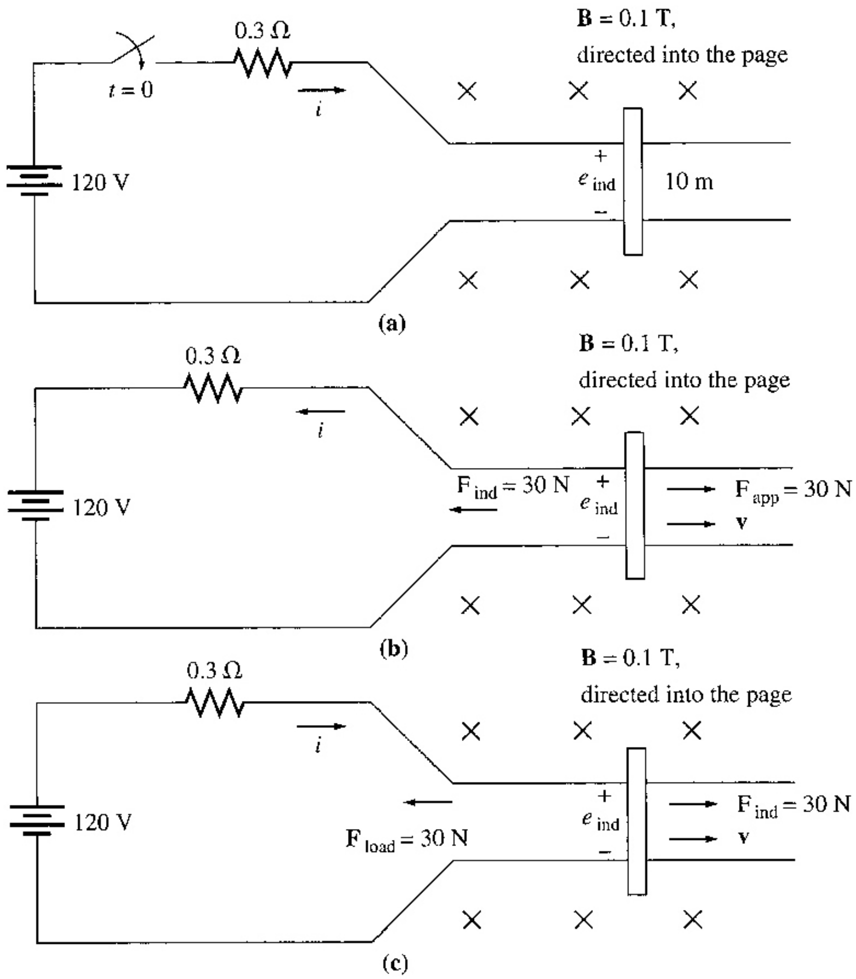
$$i_{start} = \frac{V_B}{R} = \frac{250}{0.1} = 2500A$$

4. This current is very high (10x in excess of the rated current).

5. How to prevent? → insert an extra resistance into the circuit during starting to limit current flow until e_{ind} builds up enough to limit it, as shown here:



Example 1.10



The linear dc machine is as shown in (a).

(a) What is the machine's maximum starting current? What is the steady state velocity at no load?

(b) Suppose a 30N force pointing to the right were applied to the bar (figure b). What would the steady-state speed be? How much power would the bar be producing or consuming? How much power would the bar be producing or consuming? Is the machine acting as a motor or a generator?

(c) Now suppose a 30N force pointing to the left were applied to the bar (figure c). What would the new steady-state speed be? Is the machine a motor or generator now?

CHAPTER 2 – TRANSFORMERS

Summary:

1. Types and Construction of Transformers

2. The Ideal Transformer
   - Power in an Ideal Transformer
   - Impedance transformation through a transformer
   - Analysis of circuits containing ideal transformer

3. Theory of operation of real single-phase transformers.
   - The voltage ratio across a transformer
   - The magnetization current in a Real Transformer
   - The current ratio on a transformer and the Dot Convention

4. The Equivalent Circuit of a Transformer.
   - Exact equivalent circuit
   - Approximate equivalent circuit
   - Determining the values pf components in the transformer model

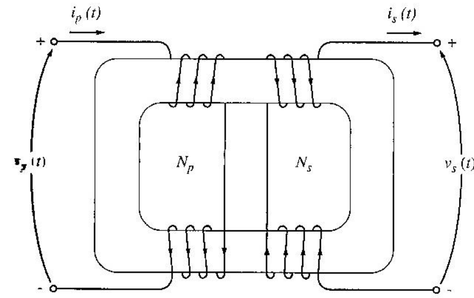
5. The Per-Unit System of Measurement

6. Transformer voltage regulation and efficiency
   - The transformer phasor diagram
   - Transformer efficiency

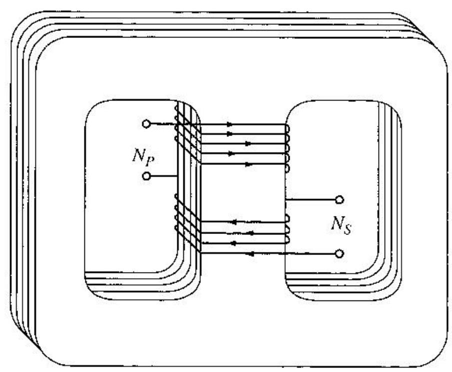
7. Three phase transformers

1. Types and Construction of Transformers

Types of cores for power transformer (both types are constructed from thin laminations electrically isolated from each other – minimize eddy currents)



i) *Core Form* : a simple rectangular laminated piece of steel with the transformer windings wrapped around two sides of the rectangle.



ii) *Shell Form* : a three legged laminated core with the windings wrapped around the centre leg.

The primary and secondary windings are wrapped one on top of the other with the low-voltage winding innermost, due to 2 purposes:

i) It simplifies the problem of insulating the high-voltage winding from the core.
ii) It results in much less leakage flux

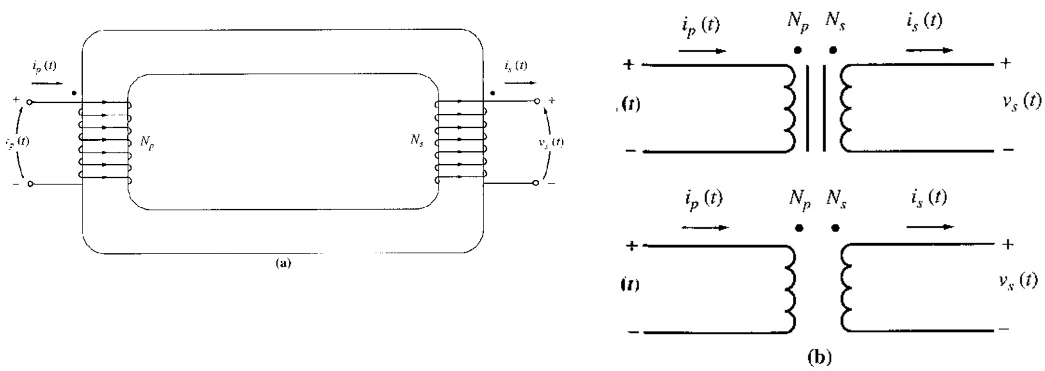
Types of transformers:

i) Step up/Unit transformers – Usually located at the output of a generator. Its function is to step up the voltage level so that transmission of power is possible.
ii) Step down/Substation transformers – Located at main distribution or secondary level transmission substations. Its function is to lower the voltage levels for distribution 1st level purposes.
iii) Distribution Transformers – located at small distribution substation. It lowers the voltage levels for 2nd level distribution purposes.
iv) Special Purpose Transformers - E.g. Potential Transformer (PT) , Current Transformer (CT)

2. The Ideal Transformer

1. Definition – a lossless device with an input winding and an output winding.

2. Figures below show an ideal transformer and schematic symbols of a transformer.



3. The transformer has N_p turns of wire on its primary side and N_s turns of wire on its secondary sides. The relationship between the primary and secondary voltage is as follows:

$$\frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$$

where *a* is the turns ratio of the transformer.

4. The relationship between primary and secondary current is:

$$N_p\, i_p\,(t) = N_s\, i_s\,(t)$$

$$\frac{i_p(t)}{i_s(t)} = \frac{1}{a}$$

5. Note that since both type of relations gives a constant ratio, hence the transformer only changes ONLY the magnitude value of current and voltage. Phase angles are not affected.

6. The dot convention in schematic diagram for transformers has the following relationship:

   i) If the primary **voltage** is +ve at the dotted end of the winding wrt the undotted end, then the secondary voltage will be positive at the dotted end also. Voltage polarities are the same wrt the dots on each side of the core.

   ii) If the primary **current** of the transformer flows **into** the dotted end of the primary winding, the secondary current will flow **out** of the dotted end of the secondary winding.

Power in an Ideal Transformer

1. The power supplied to the transformer by the primary circuit:

$$P_{in} = V_p I_p \cos \theta_p$$

   Where θ_p = the angle between the primary voltage and the primary current. The power supplied by the transformer secondary circuit to its loads is given by:

$$P_{out} = V_s I_s \cos \theta_s$$

   Where θ_s = the angle between the secondary voltage and the secondary current.

2. The primary and secondary windings of an ideal transformer have the SAME power factor – because voltage and current angles are unaffected $\theta_p - \theta_s = \theta$

3. How does power going into the primary circuit compare to the power coming out?

   $P_{out} = V_s I_s \cos \theta$

   Also, $V_s = V_p/a$ and $I_s = a\, I_p$

   So, $P_{out} = \frac{V_p}{a}(aI_p)\cos\theta$

   $P_{out} = V_p I_p \cos \theta = P_{in}$

   The same idea can be applied for reactive power Q and apparent power S.

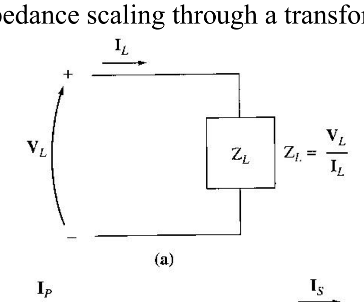
   Output power = Input power

Impedance Transformation through a Transformer

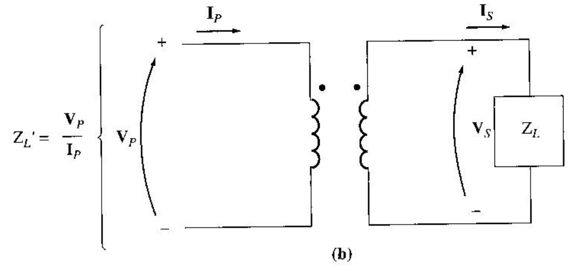
1. The impedance of a device or an element is defined as the ratio of the phasor voltage across it to the phasor current flowing through it:

$$Z_L = \frac{V_L}{I_L}$$

2. Definition of impedance and impedance scaling through a transformer:



(a)



(b)

3. Hence, the impedance of the load is: $$Z_L = \frac{V_S}{I_S}$$

4. The apparent impedance of the primary circuit of the transformer is: $$Z_L' = \frac{V_P}{I_P}$$

5. Since primary voltage can be expressed as $V_P=aV_S$, and primary current as $I_P=I_S/a$, thus the apparent impedance of the primary is

$$Z_L' = \frac{V_P}{I_P} = \frac{aV_S}{I_S / a} = a^2 \frac{V_S}{I_S}$$

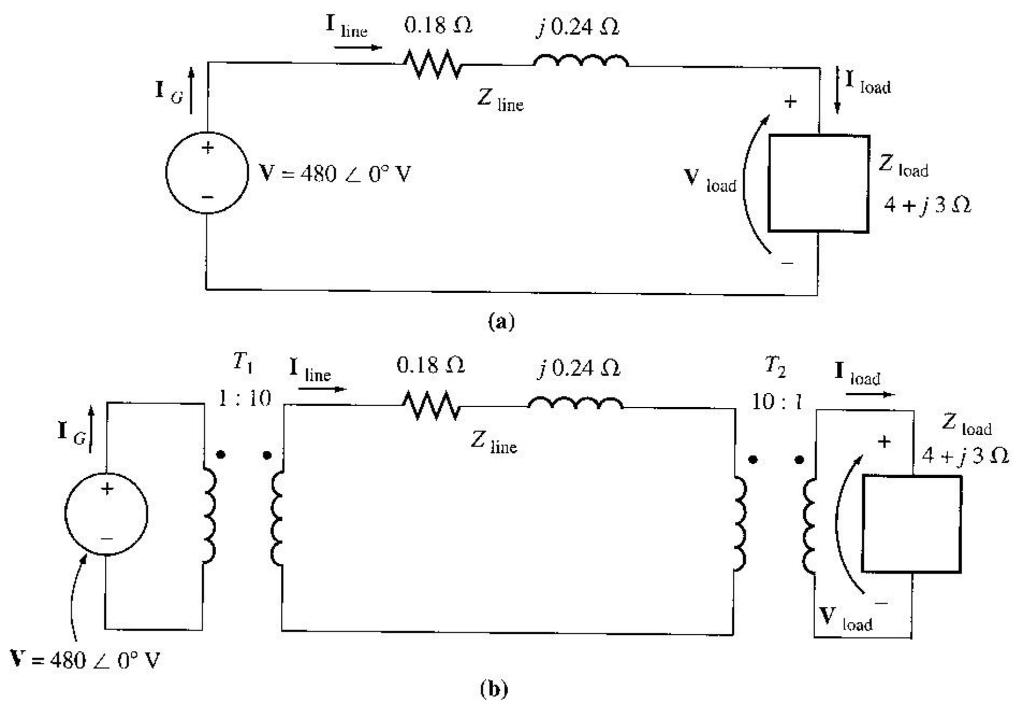
$$Z_L' = a^2 Z_L$$

Analysis of Circuits containing Ideal Transformers

The easiest way for circuit analysis that has a transformer incorporated is by simplifying the transformer into an equivalent circuit.

Example 2.1

A generator rated at 480V, 60 Hz is connected a transmission line with an impedance of 0.18+j0.24Ω. At the end of the transmission line there is a load of 4+j3Ω.



(a) If the power system is exactly as described above in Figure (a), what will the voltage at the load be? What will the transmission line losses be?
(b) Suppose a 1:10 step-up transformer is placed at the generator end of the transmission line and a 10:1 step-down transformer is placed at the load end of the line (Figure (b)). What will the load voltage be now? What will the transmission line losses be now?

3. Theory of Operation of Real Single-Phase Transformers

Ideal transformers may never exist due to the fact that there are losses associated to the operation of transformers. Hence there is a need to actually look into losses and calculation of real single phase transformers.

Assume that there is a transformer with its primary windings connected to a varying single phase voltage supply, and the output is open circuit.

Right after we activate the power supply, flux will be generated in the primary coils, based upon Faraday's law,

$$e_{ind} = \frac{d\lambda}{dt}$$

where λ is the flux linkage in the coil across which the voltage is being induced. The flux linkage λ is the sum of the flux passing through each turn in the coil added over all the turns of the coil.

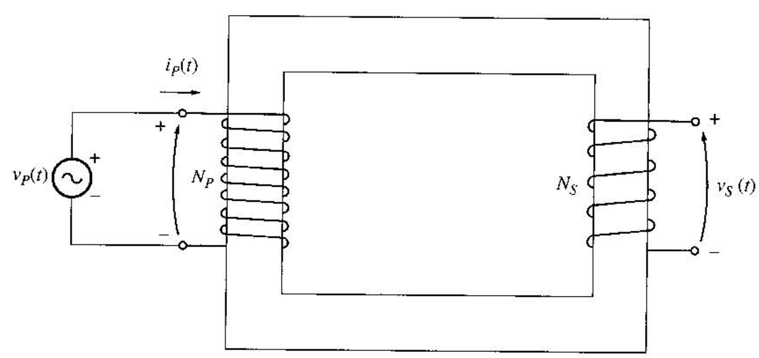
$$\lambda = \sum_{i=1}^{N} \phi_i$$

This relation is true provided on the assumption that the flux induced at each turn is at the same magnitude and direction. But in reality, the flux value at each turn may vary due to the position of the coil it self, at certain positions, there may be a higher flux level due to combination of other flux from other turns of the primary winding.

Hence the most suitable approach is to actually average the flux level as $\bar{\phi} = \frac{\lambda}{N}$

Hence Faraday's law may be rewritten as: $e_{ind} = N \frac{d\bar{\phi}}{dt}$

The voltage ratio across a Transformer



f the voltage source is $v_p(t)$, how will the transformer react to this applied voltage?
Based upon Faraday's Law, looking at the primary side of the transformer, we can determine the average flux level based upon the number of turns; where,

$$\bar{\phi} = \frac{1}{N_P} \int v_P(t)\, dt$$

This relation means that the average flux at the primary winding is proportional to the voltage level at the primary side divided by the number of turns at the primary winding. This generated flux will travel to the secondary side hence inducing potential across the secondary terminal.

For an ideal transformer, we assume that 100% of flux would travel to the secondary windings. However, in reality, there are flux which does not reach the secondary coil, in this case the flux leaks out of the transformer core into the surrounding. This leak is termed as **flux leakage**.

Taking into account the leakage flux, the flux that reaches the secondary side is termed as **mutual flux**.

Looking at the secondary side, there are similar division of flux; hence the overall picture of flux flow may be seen as below:

Primary Side:

$$\overline{\phi}_P = \phi_M + \phi_{LP}$$

$\overline{\phi}_P$ = total average primary flux

ϕ_M = flux component linking both rpimary and secondary coils

ϕ_{LP} = primary leakage flux

For the secondary side, similar division applies.

Hence, looking back at Faraday's Law,

$$v_P(t) = N_P \frac{d\overline{\phi}_P}{dt} = N_P \frac{d\overline{\phi}_M}{dt} + N_P \frac{d\overline{\phi}_{LP}}{dt}$$

Or this equation may be rewritten into:

$$v_P(t) = e_P(t) + e_{LP}(t)$$

The same may be written for the secondary voltage.

The primary voltage due to the mutual flux is given by

$$e_P(t) = N_P \frac{d\phi_M}{dt}$$

And the same goes for the secondary (just replace 'P' with 'S')

From these two relationships (primary and secondary voltage), we have

$$\frac{e_P(t)}{N_P} = \frac{d\phi_M}{dt} = \frac{e_S(t)}{N_S}$$

Therefore,

$$\frac{e_P(t)}{e_S(t)} = \frac{N_P}{N_S} = a$$

Magnetization Current in a Real transformer

Although the output of the transformer is open circuit, there will still be current flow in the primary windings. The current components may be divided into 2 components:

1) Magnetization current, i_M – current required to produce flux in the core.
2) Core-loss current, i_{h+e} – current required to compensate hysteresis and eddy current losses.

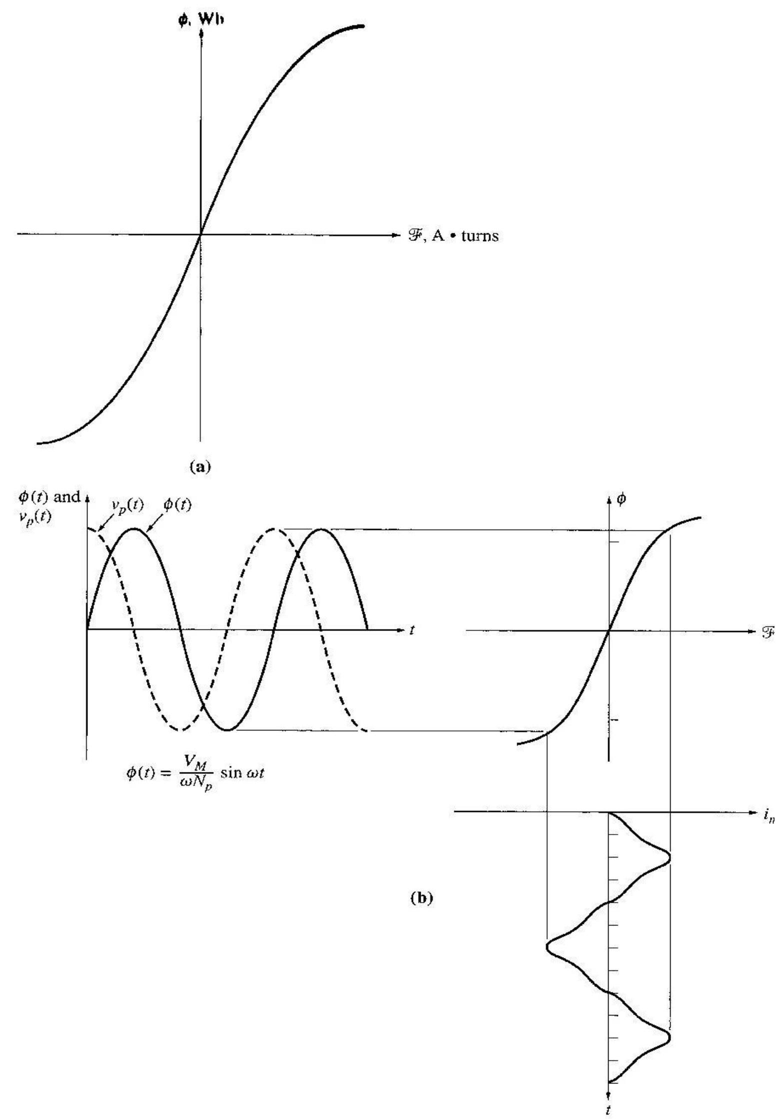
We know that the relation between current and flux is proportional since,

$$F = Ni = \phi R$$

$$\therefore i = \frac{\phi R}{N}$$

Therefore, in theory, if the flux produce in core is sinusoidal, therefore the current should also be a perfect sinusoidal. Unfortunately, this is not true since the transformer will reach to a state of near saturation at the top of the flux cycle. Hence at this point, more current is required to produce a certain amount of flux.

If the values of current required to produce a given flux are compared to the flux in the core at different times, it is possible to construct a sketch of the magnetization current in the winding on the core. This is shown below:

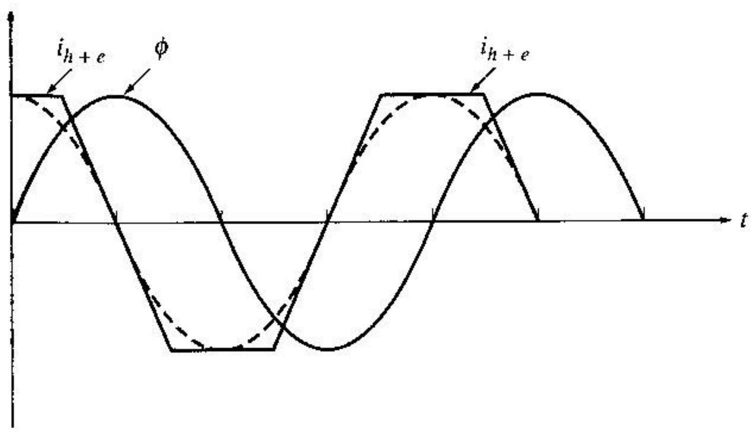


Hence we can say that current in a transformer has the following characteristics:

1. It is not sinusoidal but a combination of high frequency oscillation on top of the fundamental frequency due to magnetic saturation.
2. The current lags the voltage at 90^o
3. At saturation, the high frequency components will be extreme as such that harmonic problems will occur.

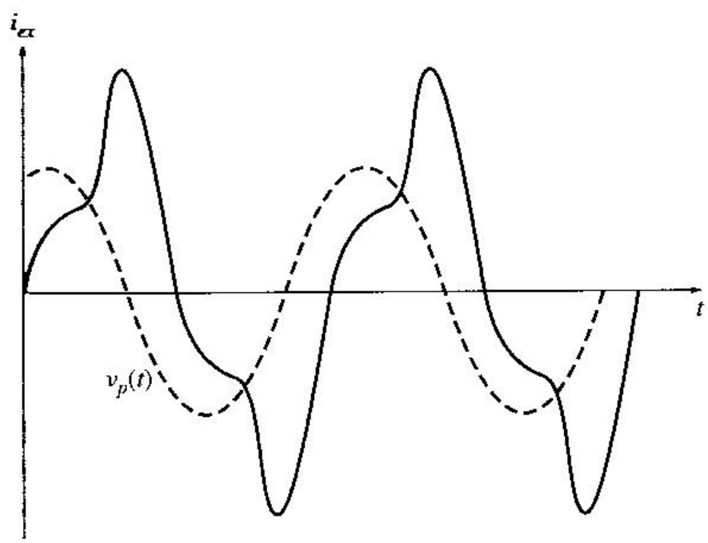
Looking at the core-loss current, it again is dependent upon hysteresis and eddy current flow. Since Eddy current is dependent upon the rate of change of flux, hence we can also say that the core-loss current is greater as the alternating flux goes past the 0 Wb. Therefore the core-loss current has the following characteristics:

a) When flux is at 0Wb, core-loss current is at a maximum hence it is in phase with the voltage applied at the primary windings.
b) Core-loss current is non-linear due to the non-linearity effects of hysteresis.



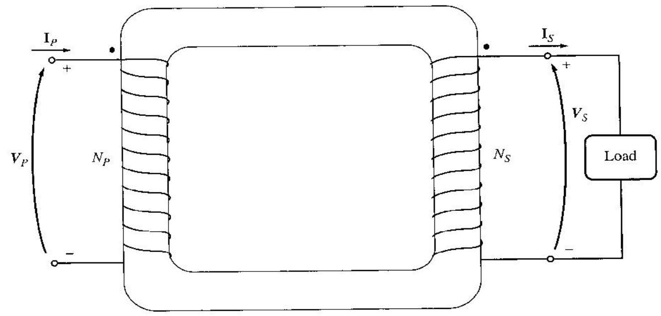
Now since that the transformer is not connected to any load, we can say that the total current flow into the primary windings is known as the **excitation current**.

$$i_{ex} = i_m + i_{h+e}$$



Current Ratio on a Transformer and the Dot Convention.

Now, a load is connected to the secondary of the transformer.



The dots help determine the polarity of the voltages and currents in the core withot having to examine physically the windings.

A current flowing into the dotted end of a winding produces a positive magnetomotive force, while a current flowing into the undotted end of a winding produces a negative magnetomotive force.

In the figure above, the net magnetomotive force is $F_{net} = N_P i_P - N_S i_S$

This net magnetomotive force must produce the net flux in the core, so

$F_{net} = N_P i_P - N_S i_S = \phi R$

Where R is the reluctance of the core. The relationship between primary and secondary current is approx

$F_{net} = N_P i_P - N_S i_S \approx 0$ as long as the core is unsaturated.

Thus,

$N_P i_P \approx N_S i_S$

$$\frac{i_P}{i_S} = \frac{N_S}{N_P} = \frac{1}{a}$$

In order for the magnetomotive force to be nearly zero, current must flow into one dotted end and out of the other dotted end.

As a conclusion, the major differences between an ideal and real transformer are as follows:

a) An ideal transformer's core does not have any hysteresis and eddy current losses.
b) The magnetization curve of an ideal transformer is similar to a step function and the net mmf is zero.
c) Flux in an ideal transformer stays in the core and hence leakage flux is zero.
d) The resistance of windings in an ideal transformer is zero.

4. The equivalent circuit of a transformer

Taking into account real transformer, there are several losses that has to be taken into account in order to accurately model the transformer, namely:

i) **Copper (I^2R) Losses** – Resistive heating losses in the primary and secondary windings of the transformer.
ii) **Eddy current Losses** – resistive heating losses in the core of the transformer. They are proportional to the square of the voltage applied to the transformer.
iii) **Hysteresis Losses** – these are associated with the rearrangement of the magnetic domains in the core during each half-cycle. They are complex, non-linear function of the voltage applied to the transformer.
iv) **Leakage flux** – The fluxes ϕ_{LP} and ϕ_{LS} which escape the core and pass through only one of the transformer windings are leakage fluxes. They then produced self-inductance in the primary and secondary coils.

The exact equivalent circuit of a real transformer

The Exact equivalent circuit will take into account all the major imperfections in real transformer.

i) Copper loss

They are modeled by placing a resistor R_P in the primary circuit and a resistor R_S in the secondary circuit.

ii) Leakage flux

As explained before, the leakage flux in the primary and secondary windings produces a voltage given by:

$$e_{LP}(t) = N_P \frac{d\phi_{LP}}{dt} \qquad e_{LS}(t) = N_S \frac{d\phi_{LS}}{dt}$$

Since flux is directly proportional to current flow, therefore we can assume that leakage flux is also proportional to current flow in the primary and secondary windings. The following may represent this proportionality:

$$\phi_{LP} = (PN_P)i_P$$

$$\phi_{LS} = (PN_S)i_S$$

Where P = permeance of flux path
N_P = number of turns on primary coils
N_S = number of turns on secondary coils

Thus,

$$e_{LP}(t) = N_P \frac{d}{dt}(PN_P)i_P = N_P{}^2 P \frac{di_P}{dt}$$

$$e_{LS}(t) = N_S \frac{d}{dt}(PN_S)i_S = N_S{}^2 P \frac{di_S}{dt}$$

The constants in these equations can be lumped together. Then,

$$e_{LP}(t) = L_P \frac{di_P}{dt}$$

$$e_{LS}(t) = L_S \frac{di_S}{dt}$$

Where $L_P = N_P^2$ P is the self-inductance of the primary coil and $L_S = N_S^2$ P is the self-inductance of the secondary coil.

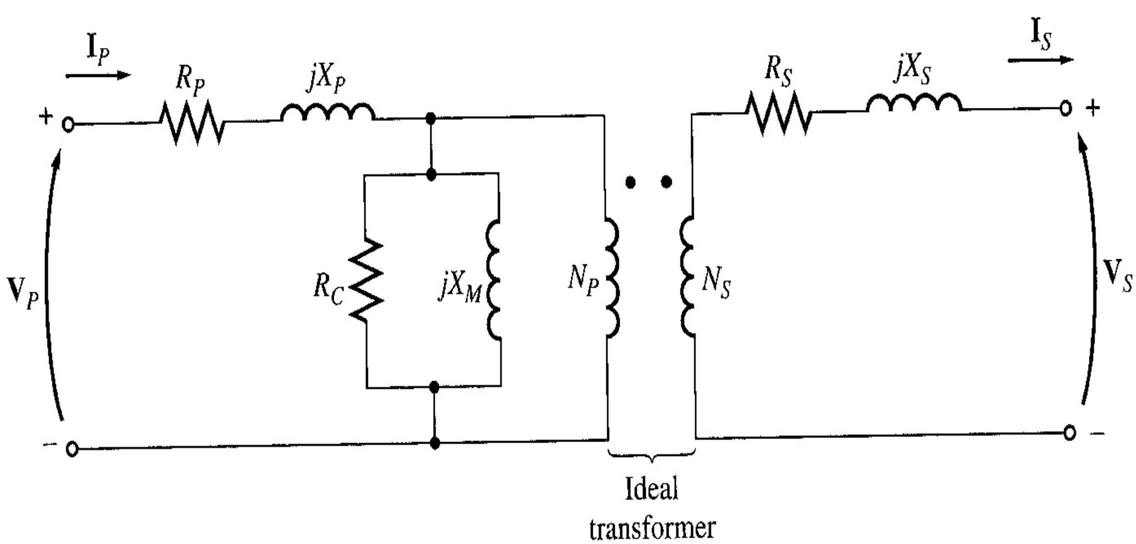
Therefore the **leakage element may be modelled as an inductance** connected together in series with the primary and secondary circuit respectively.

iii) Core excitation effects – magnetization current and hysteresis & eddy current losses

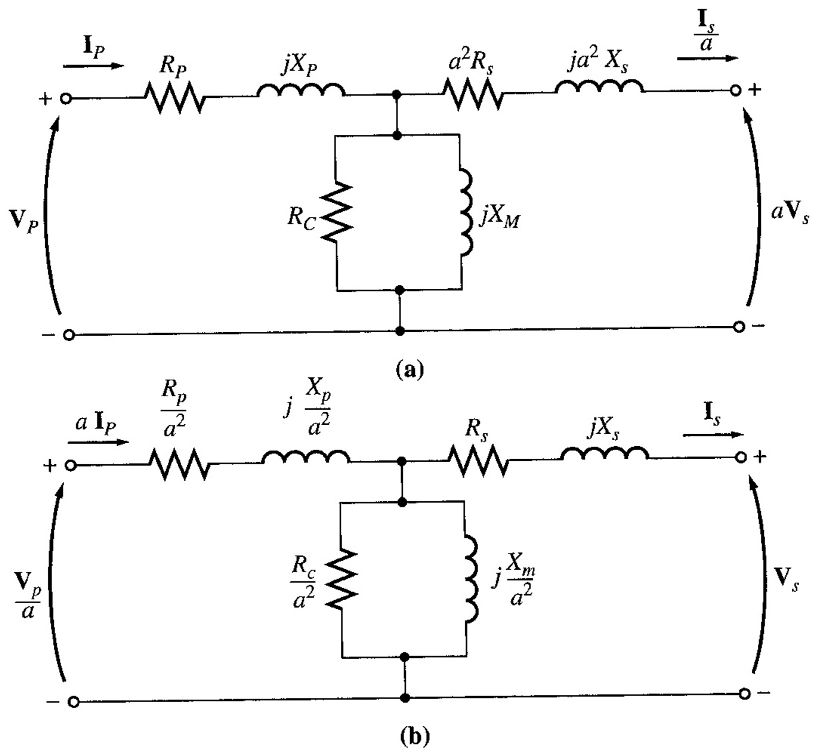
The magnetization current i_m is a current proportional (in the unsaturated region) to the voltage applied to the core and lagging the applied voltage by 90° - modeled as reactance X_m across the primary voltage source.

The core loss current i_{h+e} is a current proportional to the voltage applied to the core that is in phase with the applied voltage – modeled as a resistance R_C across the primary voltage source.

The resulting equivalent circuit:



Based upon the equivalent circuit, in order for mathematical calculation, this transformer equivalent has to be simplified by referring the impedances in the secondary back to the primary or vice versa.

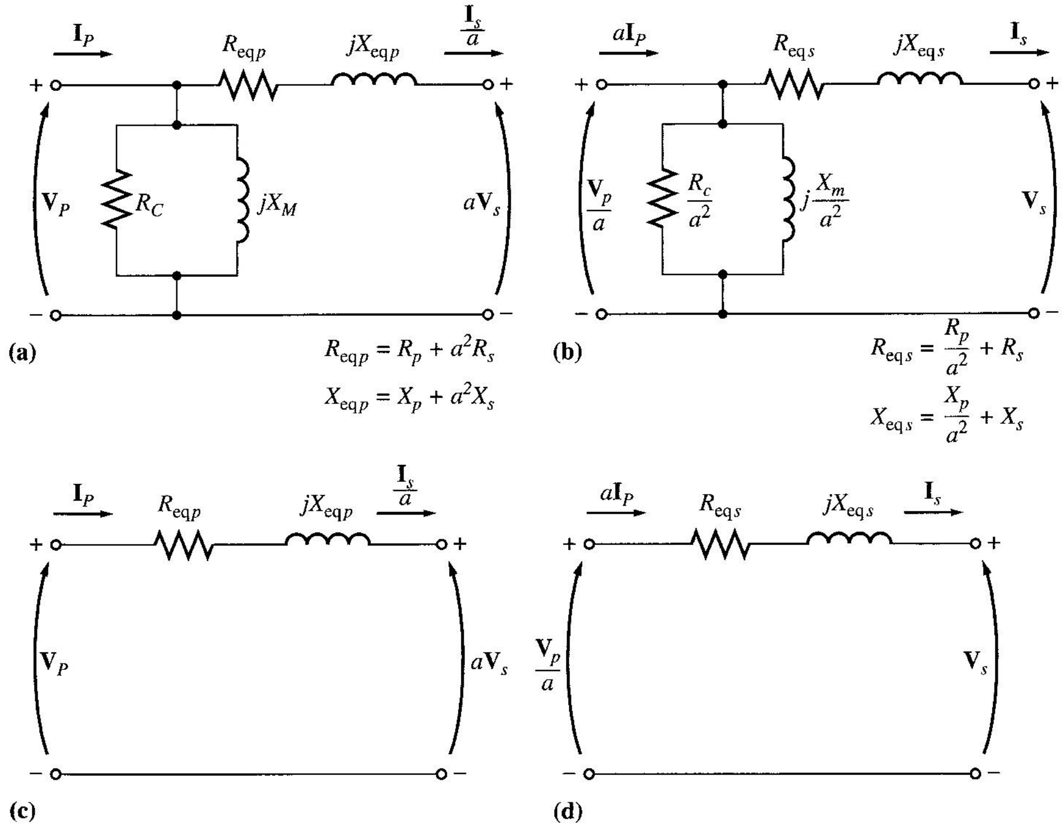


(a) Equivalent transformer circuit referring to the primary
(b) Equivalent transformer circuit referring to the secondary

Approximate Equivalent circuits of a Transformer

The derived equivalent circuit is detailed but it is considered to be too complex for practical engineering applications. The main problem in calculations will be the excitation and the eddy current and hysteresis loss representation adds an extra branch in the calculations.

In practical situations, the excitation current will be relatively small as compared to the load current, which makes the resultant voltage drop across Rp and Xp to be very small, hence Rp and Xp may be lumped together with the secondary referred impedances to form and equivalent impedance. In some cases, the excitation current is neglected entirely due to its small magnitude.



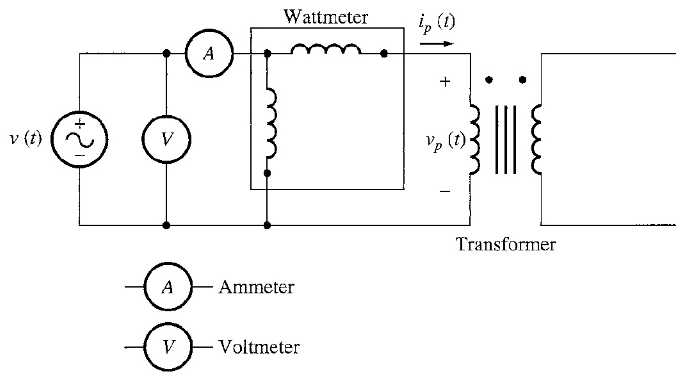
(a) Referred to the primary side
(b) Referred to the secondary side
(c) With no excitation branch, referred to the primary side
(d) With no excitation branch, referred to the secondary side

Determining the values of Components in the Transformer Model

The values of the inductances and resistances in the transformer model can be determined experimentally. An adequate approximation of these values can be obtained with the open-circuit test, and the short-circuit test.

Open-circuit Test

The transformer's secondary winding is open-circuited, and its primary winding is connected to a full-rated line voltage.



All the input current will be flowing through the excitation branch of the transformer. The series element R_P and X_P are too small in comparison to R_C and X_M to cause a significant voltage drop. Essentially all input voltage is dropped across the excitation branch.

Full line voltage is applied to the primary – input voltage, input current, input power measured. Then, power factor of the input current and magnitude and angle of the excitation impedance can be calculated.

To obtain the values of R_C and X_M , the easiest way is to find the admittance of the branch.

Conductance of the core loss resistor, $G_c = \dfrac{1}{R_C}$

Susceptance of the magnetizing inductor, $B_M = \dfrac{1}{X_M}$

These two elements are in parallel, thus their admittances add.

Total excitation admittance, $Y_E = G_C - jB_M$

$$= \frac{1}{R_C} - j\frac{1}{X_M}$$

The magnitude of the excitation admittance (referred to primary),

$$|Y_E| = \frac{I_{OC}}{V_{OC}}$$

The angle of the admittance can be found from the circuit power factor.

$$PF = \cos\theta = \frac{P_{OC}}{V_{OC} I_{OC}}$$

$$\theta = \cos^{-1}\frac{P_{OC}}{V_{OC} I_{OC}}$$

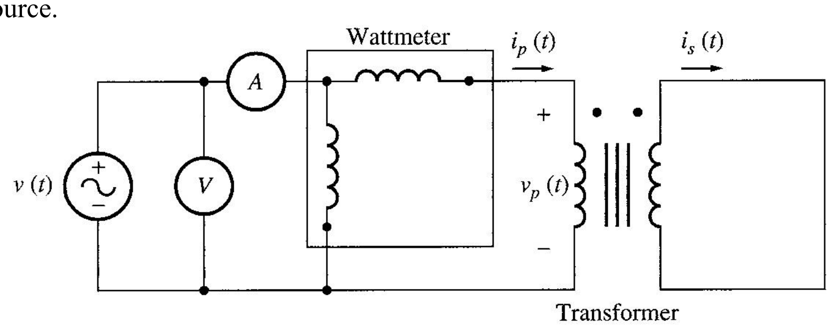
The power factor is always lagging for a real transformer. Hence,

$$Y_E = \frac{I_{OC}}{V_{OC}}\angle -\theta$$

This equation can be written in the complex number form and hence the values of R_C and X_M can be determined from the open circuit test data.

Short-circuit Test

The secondary terminals are short circuited, and the primary terminals are connected to a fairly low-voltage source.



The input voltage is adjusted until the current in the short circuited windings is equal to its rated value. The input voltage, current and power are measured.

The excitation branch is ignored, because negligible current flows through it due to low input voltage during this test. Thus, the magnitude of the series impedances referred to the primary is:

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}}$$

Power factor, PF = cos θ = $P_{SC} / V_{SC} I_{SC}$ (lagging)

Therefore,

$$Z_{SE} = \frac{V_{SC}\angle 0^\circ}{I_{SC}\angle -\theta} = \frac{V_{SC}}{I_{SC}}\angle \theta^\circ$$

The series impedance Z_{SE} $= R_{eq} + jX_{eq}$
$= (R_P + a^2 R_S) + j(X_P + a^2 X_S)$

Example 2.2

The equivalent circuit impedances of a 20kVA, 8000/240V, 60Hz transformer are to be determined. The open circuit test and the short circuit test were performed on the primary side of the transformer, and the following data were taken:

Open circuit test (primary)	Short circuit test
V_{OC} = 8000 V	V_{SC} = 489 V
I_{OC} = 0.214 A	I_{SC} = 2.5 A
P_{OC} = 400 W	P_{SC} = 240 W

Find the impedance of the approximate equivalent circuit referred to the primary side, and sketch the circuit.

5. The Per-Unit System of Measurements

The process of solving circuits containing transformers using the referring method where all the different voltage levels on different sides of the transformers are referred to a common level, can be quite tedious.

The Per-unit System of measurements eliminates this problem. The required conversions are handled automatically by the method.

In per-unit system, each electrical quantity is measured as a decimal fraction of some base level. Any quantity can be expressed on a per-unit basis by the equation

$$Quantity\ per\ unit = \frac{actual\ \ value}{base\ value\ of\ \ quantity}$$

Two base quantities are selected to define a given per-unit system. The ones usually selected are voltage and power. In a single phase system, the relationship are:

P_{base}, Q_{base} or $S_{base} = V_{base}\ I_{base}$

$$Z_{base} = \frac{V_{base}}{I_{base}} \qquad Y_{base} = \frac{I_{base}}{V_{base}}$$

And $$Z_{base} = \frac{(V_{base})^2}{S_{base}}$$

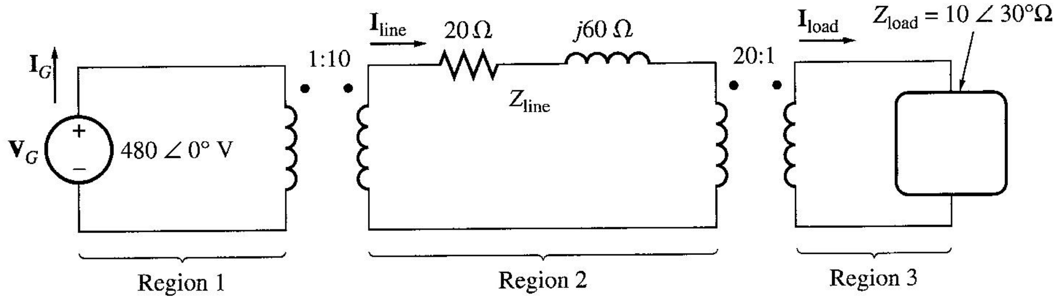
All other values can be computed once the base values of S (or P) and V have been selected.

In a power system, a base apparent power and voltage are selected at a specified point in the system. A transformer has no effect on the base apparent power of the system, since the apparent power equals the apparent power out.

Voltage changes as it goes through a transformer, so V_{base} changes at every transformer in the system according to its turns ratio. Thus, the process of referring quantities to a common level is automatically taken care of.

Example 2.3

A simple power system is shown below. This system contains a 480V generator connected to an ideal 1:10 step-up transformer, a transmission line, an ideal 20:1 step-down transformer, and a load. The impedance of the transmission line is 20 + j60Ω, and the impedance of the load is $10\angle 30°\Omega$. The base values for this system are chosen to be 480V and 10kVA at the generator.



(c) Find the base voltage, current, impedance, and apparent power at every point in the power system.
(d) Convert this system to its per-unit equivalent circuit.
(e) Find the power supplied to the load in this system.
(f) Find the power lost in the transmission line.

6. Transformer Voltage Regulation and Efficiency

The output voltage of a transformer varies with the load even if the input voltage remains constant. This is because a real transformer has series impedance within it. Full load Voltage Regulation is a quantity that compares the output voltage at no load with the output voltage at full load, defined by this equation:

$$VR = \frac{V_{S,nl} - V_{S,fl}}{V_{S,fl}} x\,100\%$$

At no load, $V_S = V_P/a$ thus,

$$VR = \frac{(V_P / a) - V_{S,fl}}{V_{S,fl}} x\,100\%$$

In per-unit system,

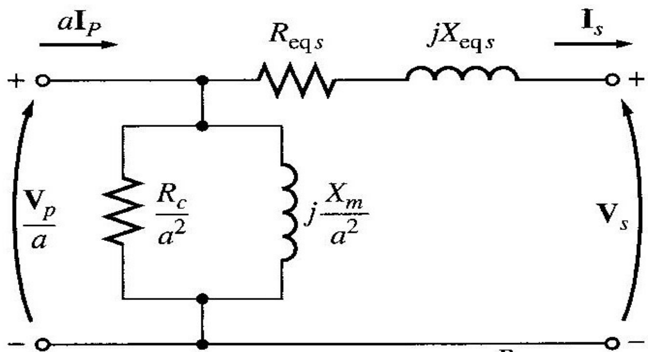
$$VR = \frac{V_{P,pu} - V_{S,fl,pu}}{V_{S,fl,pu}} x\,100\%$$

Ideal transformer, VR = 0%.

<u>The transformer phasor diagram</u>

To determine the voltage regulation of a transformer, we must understand the voltage drops within it.

Consider the simplified equivalent circuit referred to the secondary side:



(b)

$$R_{eq\,s} = \frac{R_p}{a^2} + R_s$$

$$X_{eq\,s} = \frac{X_p}{a^2} + X_s$$

Ignoring the excitation of the branch (since the current flow through the branch is considered to be small), more consideration is given to the series impedances (Req +jXeq). Voltage Regulation depends on magnitude of the series impedance and the phase angle of the current flowing through the transformer. Phasor diagrams will determine the effects of these factors on the voltage regulation. A phasor diagram consist of current and voltage vectors.

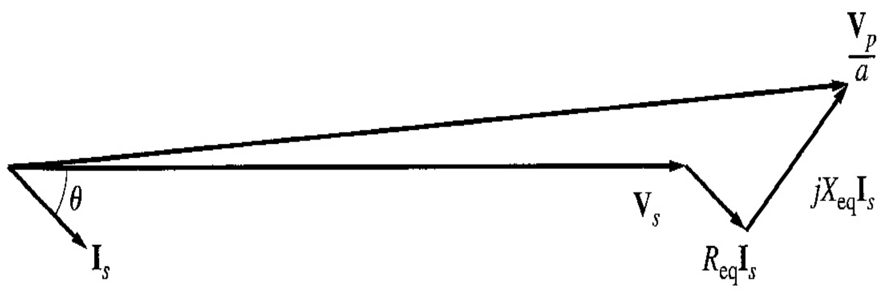
Assume that the reference phasor is the secondary voltage, V_S. Therefore the reference phasor will have 0 degrees in terms of angle.

Based upon the equivalent circuit, apply Kirchoff Voltage Law,

$$\frac{V_P}{a} = V_S + R_{eq} I_S + jX_{eq} I_S$$

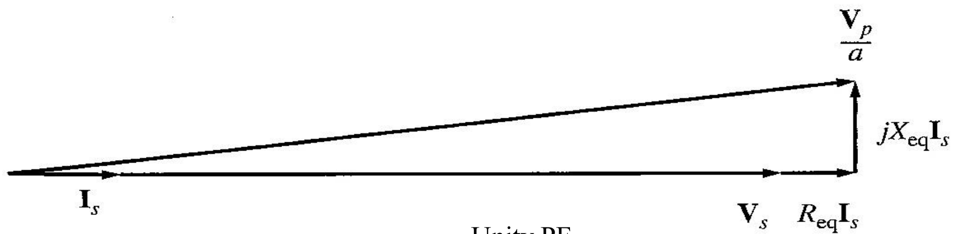
From this equation, the phasor diagram can be visualised.

Figure below shows a phasor diagram of a transformer operating at a lagging power factor. For lagging loads, $V_P / a > V_S$ so the voltage regulation with lagging loads is > 0.

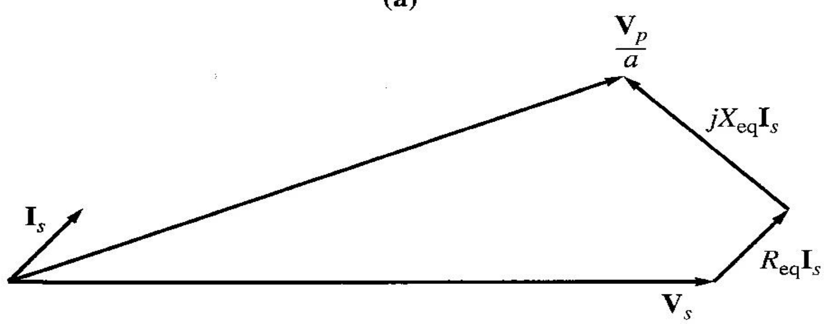


At lagging power factor

When the power factor is unity, V_S is lower than V_P so VR > 0. But, VR is smaller than before (during lagging PF).
With a leading power factor, V_S is higher than the referred V_P so VR < 0.



(a) Unity PF



(b) Leading PF

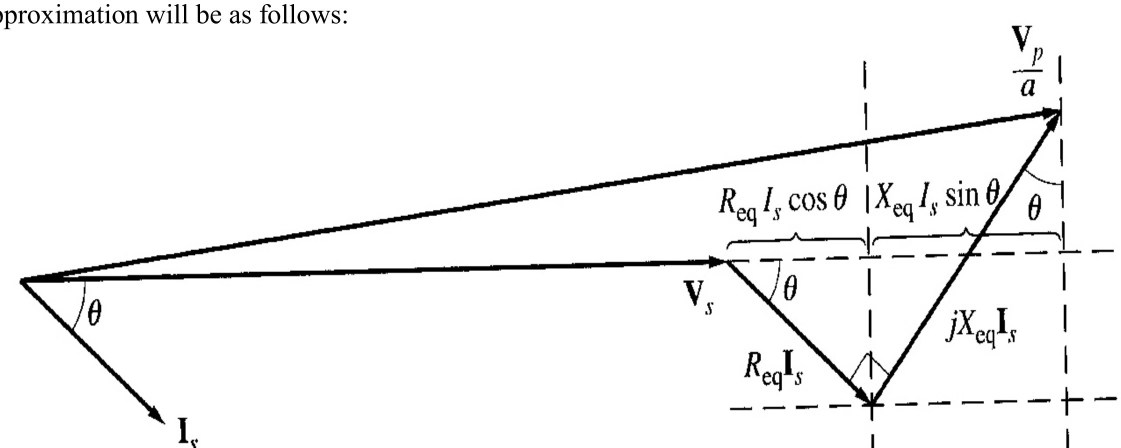
In summary:

Lagging PF	$V_P / a > V_S$	VR > 0
Unity PF	$V_P / a > V_S$	VR > 0 (smaller than VR lag)
Leading PF	$V_S > V_P / a$	VR < 0

Due to the fact that transformer is usually operated at lagging pf, hence there is an approximate method to simplify calculations.

<u>Simplified Voltage Regulation Calculation</u>

For **lagging loads**, the vertical components of R_{eq} and X_{eq} will partially cancel each other. Due to that, the angle of V_P/a will be very small, hence we can assume that V_P/a is horizontal. Therefore the approximation will be as follows:



$$\frac{V_p}{a} \approx V_s + R_{eq} I_s \cos\theta + X_{eq} I_s \sin\theta$$

Transformer Efficiency

Transformer efficiency is defined as (applies to motors, generators and transformers):

$$\eta = \frac{P_{out}}{P_{in}} x100\%$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} x100\%$$

Types of losses incurred in a transformer:

- Copper I^2R losses
- Hysteresis losses
- Eddy current losses

Therefore, for a transformer, efficiency may be calculated using the following:

$$\eta = \frac{V_S I_S \cos\theta}{P_{Cu} + P_{core} + V_S I_S \cos\theta} x100\%$$

Example 2.5

A 15kVA, 2300/230 V transformer is to be tested to determine its excitation branch components, its series impedances, and its voltage regulation. The following data have been taken from the primary side of the transformer:

Open circuit test	*Short-circuit test*
V_{OC}= 2300V	V_{SC}= 47V
I_{OC}= 0.21A	I_{SC}= 6A
P_{OC}= 50W	P_{SC}= 160W

(a) Find the equivalent circuit referred to the high voltage side
(b) Find the equivalent circuit referred to the low voltage side
(c) Calculate the full-load voltage regulation at 0.8 lagging PF, 1.0 PF, and at 0.8 leading PF.
(d) Find the efficiency at full load with PF 0.8 lagging.

7. Three phase Transformers

Transformers for 3-phase circuits can be constructed in two ways:

- connect 3 single phase transformers
- Three sets of windings wrapped around a common core.

Three-Phase Transformer Connections

The primaries and secondaries of any three-phase transformer can be independently connected in either a wye (Y) or a delta (Δ).

The important point to note when analyzing any 3-phase transformer is to look at a single transformer in the bank. **Any single phase transformer in the bank behaves exactly like the single-phase transformers already studied**.

The impedance, voltage regulation, efficiency, and similar calculations for three phase transformers are done on a **per-phase basis**, using same techniques as single-phase transformers.

A simple concept that all students must remember is that, for a Delta configuration,

$$V_{\phi P} = V_L \qquad I_{\phi P} = \frac{I_L}{\sqrt{3}} \qquad S_{\phi P} = \frac{S}{3}$$

For Wye configuration,

$$V_{\phi P} = \frac{V_L}{\sqrt{3}} \qquad I_{\phi P} = I_L \qquad S_{\phi P} = \frac{S}{3}$$

Calculating 3 phase transformer turns ratio

The basic concept of calculating the turns ratio for a single phase transformer is utilised where,

$$a = \frac{V_{\phi P}}{V_{\phi S}}$$

Therefore to cater for 3 phase transformer, suitable conversion into per phase is needed to relate the turns ratio of the transformer with the line voltages.

The Per-unit System for 3-Phase Transformer

The per unit system of measurements application for 3-phase is the same as in single phase transformers. The single-phase base equations apply to 3-phase on a per-phase basis.

Say the total base voltampere value of a transformer bank is called S_{base}, then the base voltampere value of one of the transformer is

$$S_{1\phi,base} = \frac{S_{base}}{3}$$

And the base current and impedance are

$$I_{\phi,base} = \frac{S_{1\phi,base}}{V_{\phi base}} \qquad Z_{base} = \frac{(V_{\phi,base})^2}{S_{1\phi,base}}$$

$$I_{\phi,base} = \frac{S_{base}}{3V_{\phi base}} \qquad Z_{base} = \frac{3(V_{\phi,base})^2}{S_{base}}$$

Example 2.9

A 50-kVA 13,800/208-V Δ-Y distribution transformer has a resistance of 1% and a reactance of 7% per unit.

a. What is the transformer's phase impedance referred to the high voltage side?
b. Calculate this transformer's voltage regulation at full load and 0.8PF lagging, using the calculated high side impedance.
c. Calculate this transformer's voltage regulation under the same conditions, using the per-unit system.

CHAPTER 4 – AC Machinery Fundamentals

Summary:

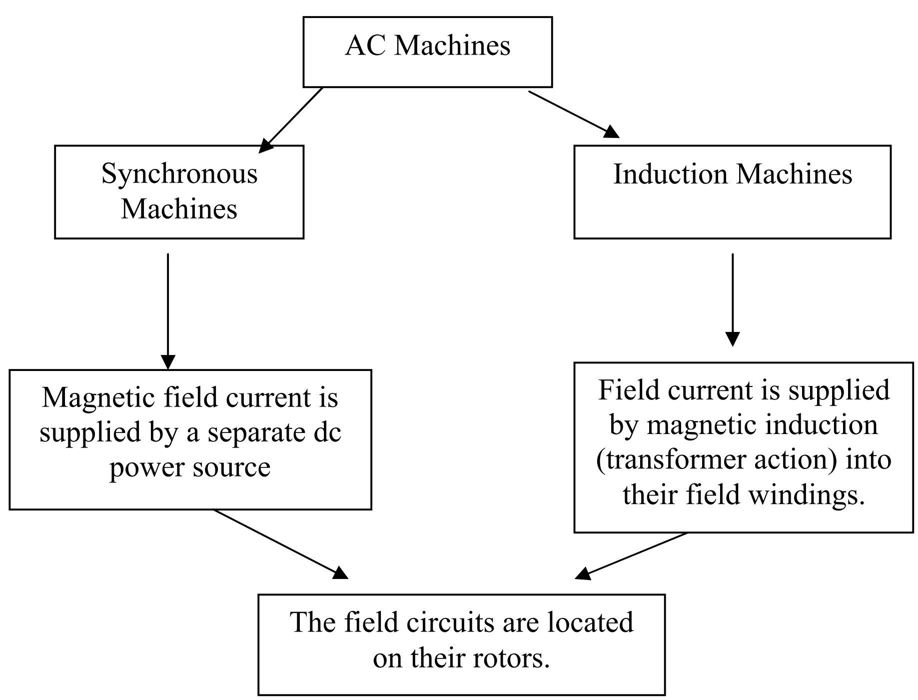
1. **A simple loop in a uniform magnetic field**
   - The voltage induced in a simple rotating loop
   - The Torque induced in a current-carrying loop

2. **The Rotating Magnetic Field**
   - Proof of the rotating magnetic field concept
   - The relationship between Electrical Frequency and the Speed of Magnetic field rotation
   - Reversing the direction of magnetic field rotation

3. **Magnetomotive Force and Flux Distribution on AC Machines**

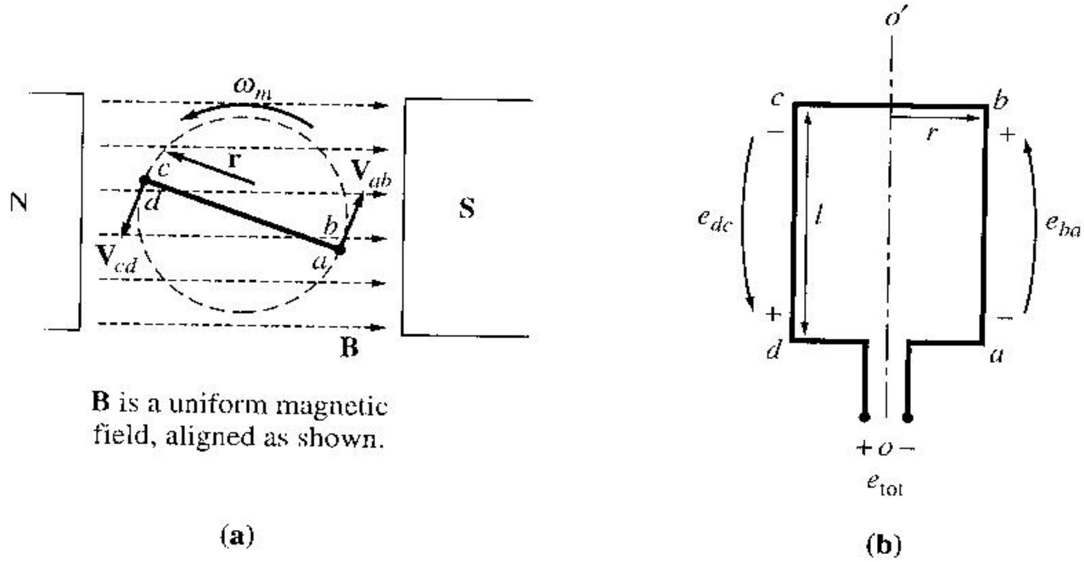
4. **Induced Voltage in AC Machines**
   - The induced voltage in a coil on a two-pole stator
   - The induced voltage in a three-phase set of coils
   - The RMS voltage in a Three-Phase Stator

5. **Induced Torque in an AC Machines**



1. A simple loop in a uniform magnetic field

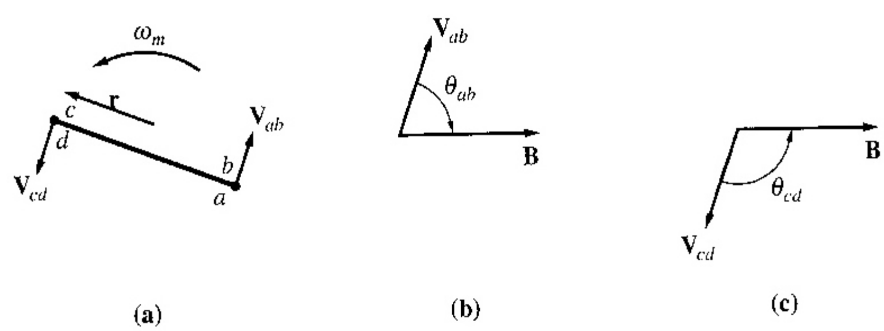
The figure below shows a simple rotating loop in a uniform magnetic field. (a) is the front view and (b) is the view of the coil. The rotating part is called the rotor, and the stationary part is called the stator.



This case in not representative of real ac machines (flux in real ac machines is not constant in either magnitude or direction). However, the factors that control the voltage and torque on the loop are the same as the factors that control the voltage and torque in real ac machines.

The voltage induced in a simple rotating loop

If the rotor (loop) is rotated, a voltage will be induced in the wire loop. To determine the magnitude and shape, examine the phasors below:



To determine the total voltage induced e_{tot} on the loop, examine each segment of the loop separately and sum all the resulting voltages. The voltage on each segment is given by equation

$$e_{ind} = (\boldsymbol{v} \times \boldsymbol{B}) \cdot l$$

(***remember that these ideas all revert back to the linear DC machine concepts in Chapter 1***).

1. Segment ***ab***

   The velocity of the wire is tangential to the path of rotation, while the magnetic field **B** points to the right. The quantity ***v x B*** points into the page, which is the same direction as segment ***ab***. Thus, the induced voltage on this segment is:

$$e_{ba} = (\boldsymbol{v} \times \boldsymbol{B}) \cdot l$$
$$= vBl \sin \theta_{ab} \quad \text{into the page}$$

2. <u>Segment *bc*</u>
   In the first half of this segment, the quantity ***v x B*** points into the page, and in the second half of this segment, the quantity ***v x B*** points out of the page. Since the length *l* is in the plane of the page, ***v x B*** is perpendicular to *l* for both portions of the segment. Thus,
$$e_{cb} = 0$$

3. <u>Segment *cd*</u>
   The velocity of the wire is tangential to the path of rotation, while **B** points to the right. The quantity ***v x B*** points into the page, which is the same direction as segment *cd*. Thus,

$$e_{cd} = (\boldsymbol{v} \times \boldsymbol{B}) \cdot l$$
$$= vBl \sin \theta_{cd} \quad \text{out of the page}$$

4. <u>Segment *da*</u>
   same as segment *bc*, ***v x B*** is perpendicular to *l*. Thus,
$$e_{da} = 0$$

Total induced voltage on the loop e_{ind}

$$= e_{ba} + e_{cb} + e_{dc} + e_{ad}$$
$$= vBl \sin \theta_{ab} + vBl \sin \theta_{cd}$$
$$= 2\, vBL \sin\theta$$

since $\theta_{ab} = 180^o - \theta_{cd}$ and $\sin \theta = \sin (180^o - \theta)$

Alternative way to express e_{ind}:

If the loop is rotating at a constant angular velocity ω, then the angle θ of the loop will increase linearly with time.

$$\theta = \omega t$$

also, the tangential velocity v of the edges of the loop is:

$$v = r\,\omega$$

where r is the radius from axis of rotation out to the edge of the loop and ω is the angular velocity of the loop. Hence,

$$e_{ind} = 2r\,\omega Bl \sin \omega t$$

since area, A = 2rl,

$$e_{ind} = AB\omega \sin \omega t$$

Finally, since maximum flux through the loop occurs when the loop is perpendicular to the magnetic flux density lines, so

$$\phi_{max} = AB$$

Thus,

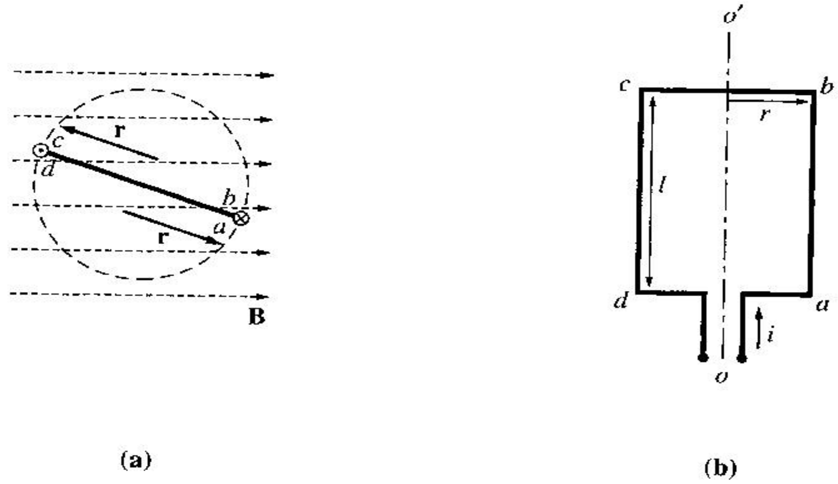
$$e_{ind} = \phi_{max}\,\omega \sin \omega t$$

From here we may conclude that the induced voltage is dependent upon:

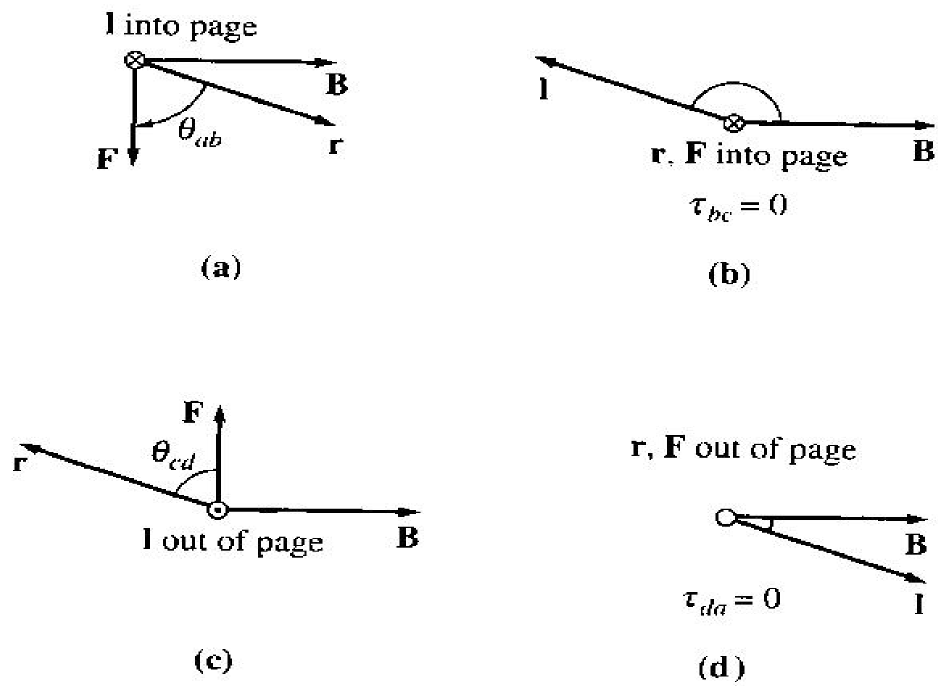
- Flux level (**the B component**)
- Speed of Rotation (**the v component**)
- Machine Constants (**the l component and machine materials**)

The Torque Induced in a Current-Carrying Loop

Assume that the rotor loop is at some arbitrary angle θ wrt the magnetic field, and that current is flowing in the loop.



To determine the magnitude and direction of the torque, examine the phasors below:



The force on each segment of the loop is given by:

$$\mathbf{F} = i\,(\boldsymbol{l} \times \boldsymbol{B})$$

$$\tau = rF\sin\theta$$

Torque on that segment,

1. <u>Segment *ab*</u>
   The direction of the current is into the page, while the magnetic field B points to the right. (***l x B***) points down. Thus,

$$\mathbf{F} = i\,(\boldsymbol{l} \times \boldsymbol{B})$$
$$= \boldsymbol{ilB} \text{ down}$$

   Resulting torque,
$$\tau_{ab} = (F)(r \sin \theta_{ab})$$
$$= rilB \sin \theta_{ab} \quad \text{clockwise}$$

2. <u>Segment *bc*</u>
   The direction of the current is in the plane of the page, while the magnetic field B points to the right. (***l x B***) points into the page. Thus,

$$\mathbf{F} = i\,(\boldsymbol{l} \times \boldsymbol{B})$$
$$= \boldsymbol{ilB} \text{ into the page}$$

   Resulting torque is zero, since vector r and l are parallel and the angle θ_{bc} is 0.

$$\tau_{bc} = (F)(r \sin \theta_{ab})$$
$$= 0$$

3. <u>Segment *cd*</u>
   The direction of the current is out of the page, while the magnetic field B points to the right. (***l x B***) points up. Thus,

$$\mathbf{F} = i\,(\boldsymbol{l} \times \boldsymbol{B})$$
$$= \boldsymbol{ilB} \text{ up}$$

   Resulting torque,
$$\tau_{cd} = (F)(r \sin \theta_{cd})$$
$$= rilB \sin \theta_{cd} \quad \text{clockwise}$$

4. <u>Segment *da*</u>
   The direction of the current is in the plane of the page, while the magnetic field B points to the right. (***l x B***) points out of the page. Thus,

$$\mathbf{F} = i\,(\boldsymbol{l} \times \boldsymbol{B})$$
$$= \boldsymbol{ilB} \text{ out of the page}$$

   Resulting torque is zero, since vector r and l are parallel and the angle θ_{da} is 0.

$$\tau_{da} = (F)(r \sin \theta_{da})$$
$$= 0$$

The total induced torque on the loop:

$$\tau_{ind} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da}$$
$$= rilB \sin \theta_{ab} + rilB \sin \theta_{cd}$$
$$= rilB \sin \theta$$

Note : the torque is maximum when the plane of the loop is parallel to the magnetic field, and the torque is zero when the plane of the loop is perpendicular to the magnetic field.

An alternative way to express the torque equation can be done which clearly relates the behaviour of the single loop to the behaviour of larger ac machines. Examine the phasors below:

If the current in the loop is as shown, that current will generate a magnetic flux density B_{loop} with the direction shown. The magnitude of B_{loop} is:

$$B_{loop} = \frac{\mu i}{G}$$

Where G is a factor that depends on the geometry of the loop.

The area of the loop A is $2rl$ and substituting these two equations into the torque equation earlier yields:

$$\tau_{ind} = \frac{AG}{\mu} B_{loop} B_S \sin\theta$$
$$= kB_{loop} B_S \sin\theta$$

Where k=AG/μ is a factor depending on the construction of the machine, B_S is used for the stator magnetic field to distinguish it from the magnetic field generated by the rotor, and θ is the angle between B_{loop} and B_S.

Thus, $$\tau_{ind} = kB_{loop} x B_S$$

From here, we may conclude that torque is dependent upon:

- Strength of rotor magnetic field
- Strength of stator magnetic field
- Angle between the 2 fields
- Machine constants

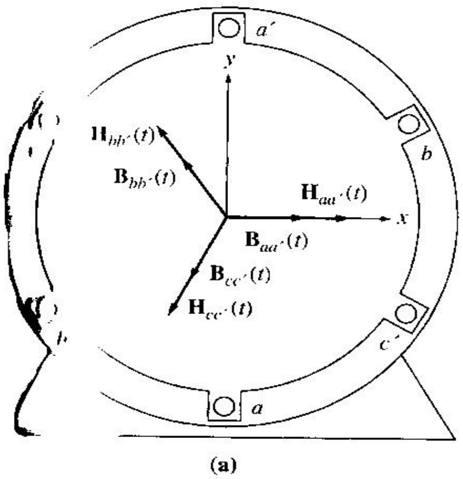
2. The Rotating Magnetic Field

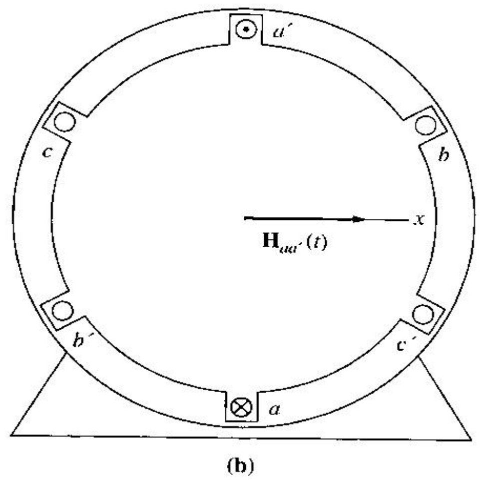
Before we have looked at how if two magnetic fields are present in a machine, then a torque will be created which will tend to line up the two magnetic fields. If one magnetic field is produced by the stator of an ac machine and the other by the rotor, then a torque will be induced in the rotor which will cause the rotor to turn and align itself with the stator magnetic field.

If there were some way to make the stator magnetic field rotate, then the induced torque in the rotor would cause it to 'chase' the stator magnetic field.

How do we make the stator magnetic field to rotate?

Fundamental principle – *a 3-phase set of currents, each of equal magnitude and differing in phase by 120º, flows in a 3-phase winding, then it will produce a rotating magnetic field of constant magnitude.* The rotating magnetic field concept is illustrated below – empty stator containing 3 coils 120º apart. It is a 2-pole winding (one north and one south).





(a) A simple three phase stator. Currents in this stator are assumed positive if they flow into the unprimed end and out the primed end of the coils.

(b) The magnetizing intensity vector $H_{aa'}(t)$ produced by a current flowing in coil aa'.

Let's apply a set of currents to the stator above and see what happens at specific instants of time. Assume currents in the 3 coils are:

$$i_{aa'}(t) = I_M \sin \omega t \; A$$

$$i_{bb'}(t) = I_M \sin(\omega t - 120°) \; A$$

$$i_{cc'}(t) = I_M \sin(\omega t - 240°) \; A$$

The current in coil aa' flows into the a end of the coil and out the a' end of the coil. It produces the magnetic field intensity:

$$H_{aa'}(t) = H_M \sin \omega t \angle 0° \; A \bullet turns \; / m$$

$$H_{bb'}(t) = H_M \sin(\omega t - 120°) \angle 120° \; A \bullet turns \; / m$$

$$H_{cc'}(t) = H_M \sin(\omega t - 240°) \angle 240° \; A \bullet turns \; / m$$

The flux densities equations are:

$$B_{aa'}(t) = B_M \sin \omega t \angle 0° \; T$$

$$B_{bb'}(t) = B_M \sin(\omega t - 120°) \angle 120° \; T$$

$$B_{cc'}(t) = B_M \sin(\omega t - 240°) \angle 240° \; T$$

Where $B_M = \mu H_M$.

At time $\omega t = 0°$

$$B_{aa'} = 0$$
$$B_{bb'} = B_M \sin(-120°) \angle 120° \; T$$
$$B_{cc'} = B_M \sin(-240°) \angle 240° \; T$$

The total magnetic field from all three coils added together will be

$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'}$$
$$= 0 + \left(-\frac{\sqrt{3}}{2}B_M\right)\angle 120° + \left(-\frac{\sqrt{3}}{2}B_M\right)\angle 240°$$
$$= 1.5B_M \angle -90°$$

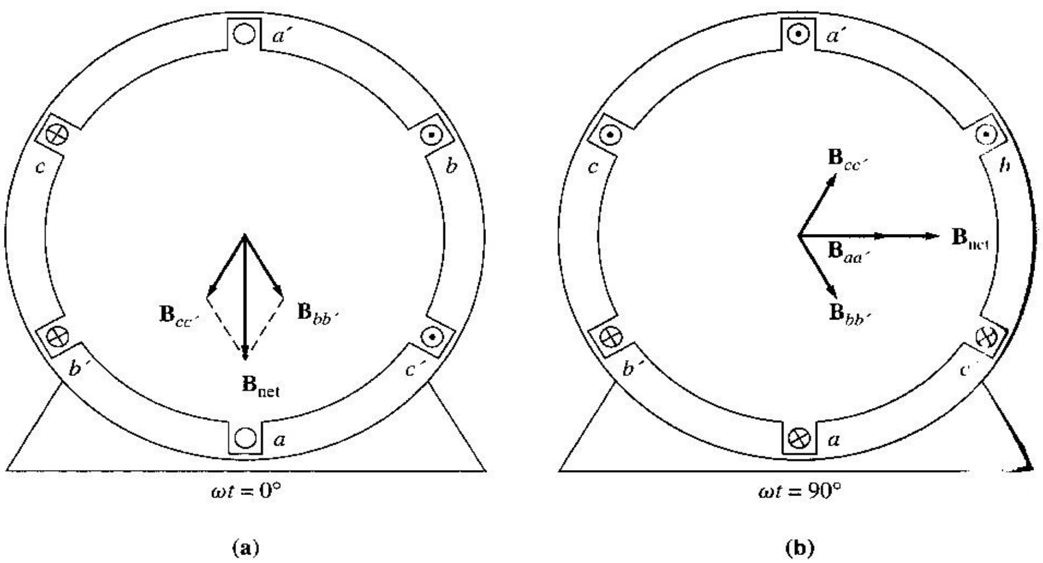
At time $\omega t = 90°$

$$B_{aa'} = B_M \angle 0°$$
$$B_{bb'} = -0.5B_M \angle 120°\ T$$
$$B_{cc'} = -0.5B_M \angle 240°\ T$$

The total magnetic field from all three coils added together will be

$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'}$$
$$= B_M \angle 0 + (-0.5B_M)\angle 120° + (-0.5B_M)\angle 240°$$
$$= 1.5B_M \angle 0°$$

The resulting magnetic flux is as shown below:



$\omega t = 0°$ $\omega t = 90°$

Proof of Rotating Magnetic Field Concept

At any time t, the magnetic field will have the same magnitude 1.5 B_M and it will continue to rotate at angular velocity ω.

Proof:
Refer again to the stator in Figure 4.1. *x* direction is to the right and *y* direction is upward.
Assume that we represent the direction of the magnetic field densities in the form of:

$$\hat{x} = horizontal \quad unit \;\; vector$$
$$\hat{y} = vertical \quad unit \;\; vector$$

To find the total magnetic flux density in the stator, simply add vectorially the three component magnetic fields and determine their sum.

We know that:

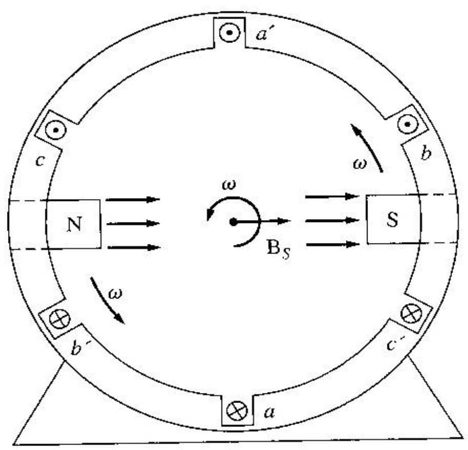
$$B_{net}(t) = B_M \sin \omega t \angle 0° + B_M \sin(\omega t - 120°)\angle 120° + B_M \sin(\omega t - 240°)\angle 240° \; T$$

We may convert the total flux density into unit vector forms to give:

$$B_{net}(t) = (1.5 B_M \sin \omega t)\hat{x} - (1.5 B_M \cos \omega t)\hat{y}$$

Notice that the magnitude of the field is a constant $1.5B_M$ and the angle changes continually in a counterclockwise direction at angular velocity ω. Also, at ωt=0°, $B_{net}=1.5B_M$ -90°, and at ωt=90°, $B_{net}=1.5B_M$ 0°.

The Relationship between Electrical Frequency and the Speed of Magnetic Field Rotation



tator

The figure above shows that the rotating magnetic field in this stator can be represented as a north pole (the flux leaves the stator) and a south pole (flux enters the stator).

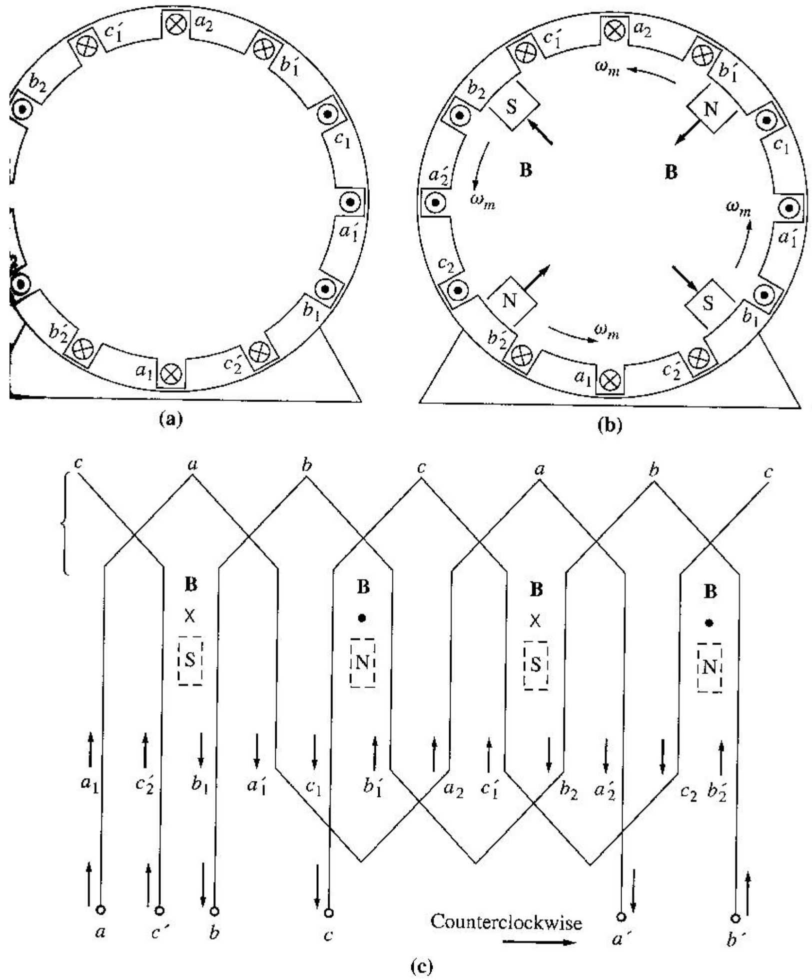
These magnetic poles complete one mechanical rotation around the stator surface for each electrical cycle of the applied current. The mechanical speed of rotation of the magnetic field in revolutions per second is equal to electric frequency in hertz:

f_e (hertz) = f_m (revolutions per second) two poles
ω_e (radians per second) = ω_m (radians per second) two poles

The windings on the 2-pole stator above occur in the order a – c' – b – a' – c – b'

If we were to double the amount of windings, hence the sequence of windings will be as follows:
a1 – c1' – b1 – a1' – c1 – b1' – a2 – c2' – b2 – a2' – c2 – b2'

For a three-phase set of currents, this stator will have 2 north poles and 2 south poles produced in the stator winding, (refer figure (b) below):



(a) A simple four-pole stator winding. (b) The resulting stator magnetic poles. Notice that there are moving poles of alternating polarity every 90° around the stator surface. (c) a winding diagram of the stator as seen from its inner surface, showing how the stator currents produce north and south magnetic poles.

In this winding, a pole moves only halfway around the stator surface in one electrical cycle. Since one electrical cycle is 360 electrical degrees, and mechanical motion is 180 mechanical degrees, the relationship between the electrical angle θ_e and the mechanical θ_m in this stator is

$$\theta_e = 2\ \theta_m$$

Thus, for a four pole winding, the electrical frequency of the current is twice the mechanical frequency of rotation:

$$f_e = 2\ f_m$$
$$\omega_e = 2\ \omega_m$$

Therefore the general format will be as follows:

$$\theta_e = \frac{P}{2}\theta_m$$

$$f_e = \frac{P}{2} f_m$$

$$\omega_e = \frac{P}{2}\omega_m$$

Also,

$$since \quad f_m = \frac{n_m}{60} \quad where\ n\ is\ the\ number\ of\ rotation$$

$$\therefore f_e = \frac{n_m}{120} P$$

Reversing the direction of Magnetic Field Rotation

If the current in any two of the 3 coils is swapped, the direction of the magnetic field's rotation will be reversed.

To prove this, phases B and C are switched:

$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'}$$

$$B_{net}(t) = B_M \sin \omega t \angle 0° + B_M \sin(\omega t - 240°)\angle 120° + B_M \sin(\omega t - 120°)\angle 240°\ T$$

Converting it into x and y unit vectors will give:

$$B_{total}(t) = (1.5B_m \sin \omega t)\hat{x} + (1.5B_m \cos \omega t)\hat{y}$$

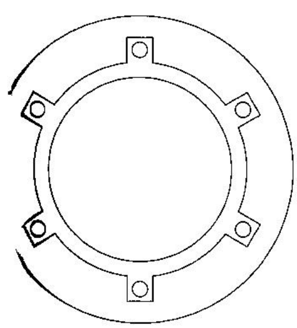
Note the +ve sign, which shows the change of magnetic field rotation.

3. Magnetomotive Force and Flux Distribution on AC Machines

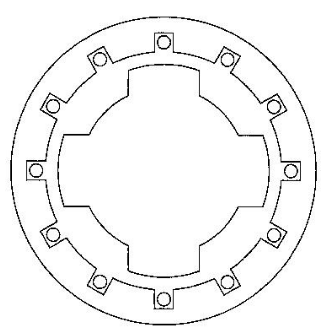
Assumptions:

- Flux produced inside an ac machine is in free space
- Direction of flux density produced by a coil of wire is perpendicular to the plane of the coil
- Direction of flux given by the right hand rule.

However, the flux in a real machine does not follow these assumptions, since there is a ferromagnetic rotor in the centre of the machine with a small air gap between the rotor and the stator. The rotor can be cylindrical (a) (nonsalient-pole), or it can have pole faces projecting out from its surface (b) (salient pole).



(a)

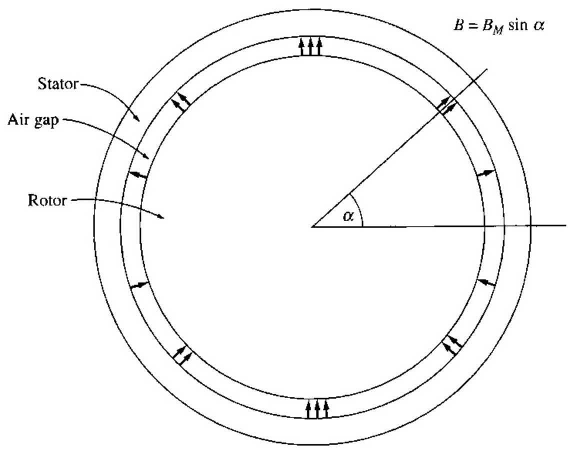


(b)

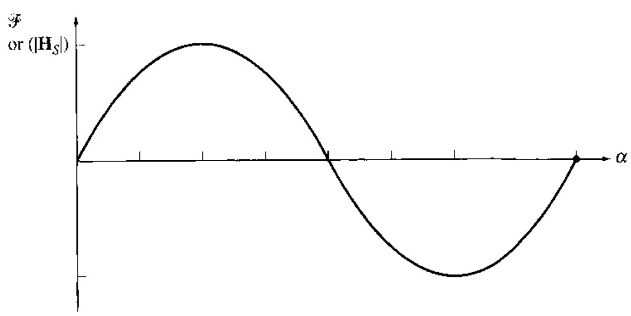
In this course, the discussion will be restricted to machines with cylindrical rotors.

The reluctance of the air gap in this machine is much higher than the reluctances of either the rotor or the stator, so the flux density vector **B** takes the shortest possible path across the air gap and jumps perpendicularly between the rotor and the stator.

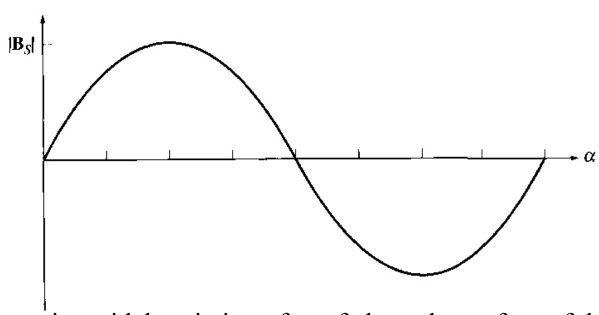
To produce a sinusoidal voltage in a machine like this, the magnitude of the flux density vector **B** must vary in a sinusoidal manner along the surface of the air gap. The flux density will vary sinusoidally only if the magnetizing intensity **H** (and mmf) varies in a sinusoidal manner along the surface of the air gap.



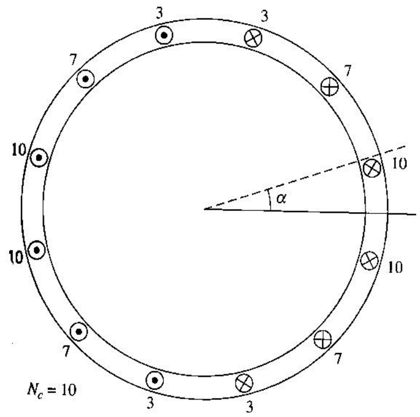
A cylindrical rotor with sinusoidally varying air-gap flux density



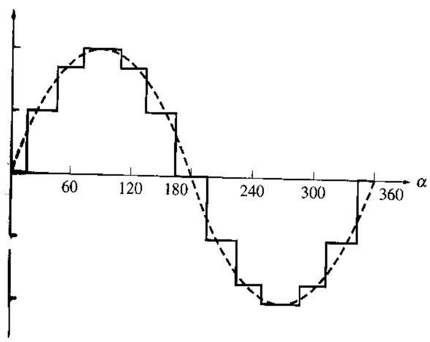
The mmf or H or B as a function of angle in the air gap



To achieve a sinusoidal variation of mmf along the surface of the air gap is to distribute the turns of the winding that produces the mmf in closely spaced slots around the surface of the machine and to vary the number of conductors in each slots in a sinusoidal manner.



An ac machine with a distributed stator winding designed to produce a sinusoidally varying air gap flux density. The number of conductors in each slot is indicated in the diagram.



The mmf distribution resulting from the winding, compared to an ideal transformer.

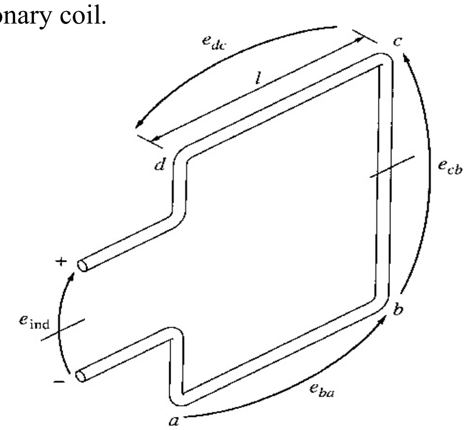
The number of conductors in each slot is $n_C = N_C \cos \alpha$ where N_C is the number of conductors at an angle of 0 degree. The distribution of conductors produces a close approximation to a sinusoidal distribution of mmf. The more slots there are and the more closely spaced the slots are, the better this approximation becomes.

In practice, it is not possible to distribute windings exactly as in the n_C equation above, since there are only a finite number of slots in a real machine and since only integral numbers of conductors can be included in each slot.

4. Induced Voltage in AC Machines

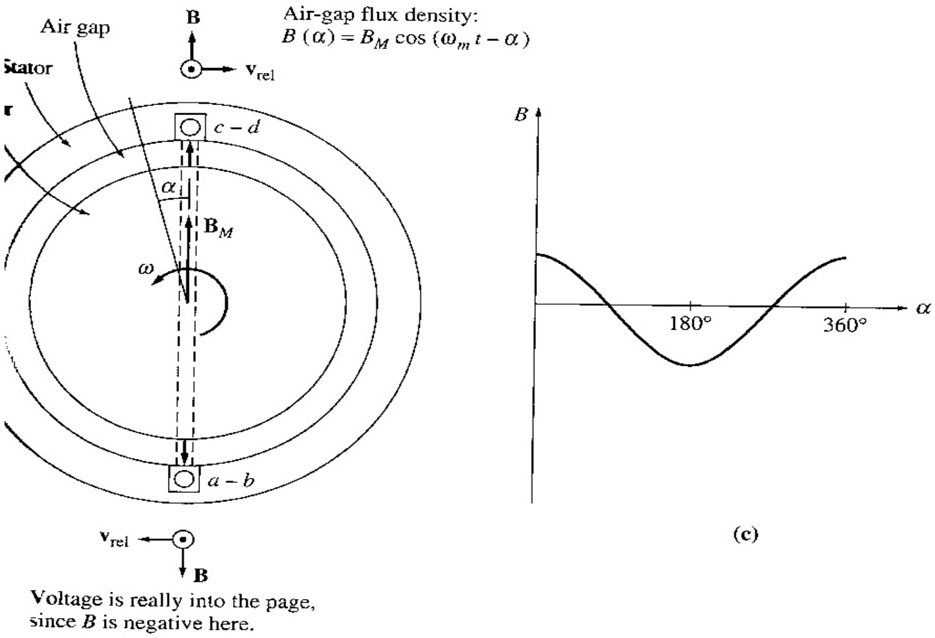
The induced voltage in a Coil on a Two-Pole Stator

Previously, discussions were made related to induced 3 phase currents producing a rotating magnetic field. **Now, lets look into the fact that a rotating magnetic field may produce voltages in the stator.** The Figures below show a rotating rotor with a sinusoidally distributed magnetic field in the centre of a stationary coil.



(a)

A rotating rotor magnetic field inside a stationary stator coil



The vector magnetic flux densities and velocities on the sides of the coil.

The flux density distribution in the air gap.

Assume that the magnetic of the flux density vector B in the air gap between the rotor and the stator varies sinusoidally with mechanical angle, while the direction of B is always radially outward. The magnitude of the flux density vector B at a point around the rotor is given by:

$$B = B_M \cos \alpha$$

Note that α is the angle between the maximum flux density (B_m) and the current magnetic flux density phasor B. Since the rotor is itself rotating within the stator at an angular velocity ω_m the magnitude of the flux density vector B at any angle α around the stator is given by:

$$B = B_M \cos (\omega t - \alpha)$$

Induced voltage in a wire is $e = (\mathbf{v} \times \mathbf{B})\, \boldsymbol{l}$

However, this equation was derived for the case of a moving wire in a stationary magnetic field. In this case, the wire is stationary and the magnetic field is moving, so the equation for induced voltage does not directly apply. Hence, we need to assume that we are "sitting on the magnetic field" so that the magnetic field appears to be stationary, and the sides of the coil will appear to go by at an apparent velocity v_{rel} and the equation can be applied.

The total voltage induced in the coil will be the sum of the voltages induced in each of its four sides. These are determined as follows:

1. ***Segment ab***

$\alpha = 180^o$. Assume that **B** is directed radially outward from the rotor, the angle between **v** and **B** in segment *ab* is 90°, while **v** x **B** is in the direction of ***l***, so

$$
\begin{aligned}
e_{ba} &= (vxB) \bullet l \\
&= vBl \quad \text{Directed out of the page} \\
&= -v[B_M \cos(\omega_m t - 180°)]l \\
&= -vB_M l \cos(\omega_m t - 180°)
\end{aligned}
$$

Where the minus sign comes from the fact that the voltage is built up with a polarity opposite to the assumed polarity.

2. ***Segment bc***
The voltage is zero, since the vector quantity **v x B** is perpendicular to **l**.

3. ***Segment cd***
$\alpha = 0^o$
Assume that **B** is directed radially outward from the rotor, the angle between **v** and **B** in segment *cd* is 90º, while **v** x **B** is in the direction of ***l***, so

$$\begin{aligned} e_{ba} &= (vxB) \bullet l \\ &= vBl \quad \text{Directed out of the page} \\ &= v(B_M \cos \omega_m t) l \\ &= vB_M l \cos \omega_m t \end{aligned}$$

4. ***Segment da***
The voltage is zero, since the vector quantity **v x B** is perpendicular to **l**.

Therefore total induced voltage:

$$e_{induced} = e_{ba} + e_{dc} = 2VB_M l \cos \omega_m t$$

Since, $v = r\omega_m$

Therefore,

$$e_{induced} = 2rlB_M \omega_m \cos \omega_m t$$

Since, $\phi = 2rlB_m$
And the angular mechanical velocity should be equal to the angular electrical velocity,

$$e_{induced} = \phi\omega \cos \omega t$$

or (taking into account number of turns of windings),

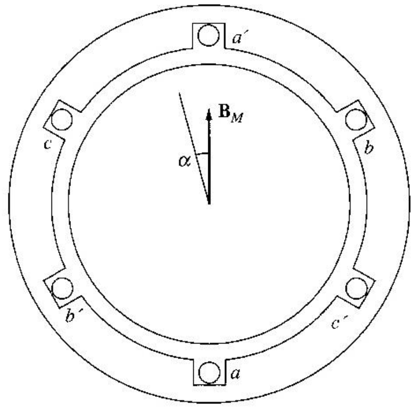
$$e_{induced} = N_c \phi\omega \cos \omega t$$

Remember: This derivation goes through the induced voltage in the stator when there is a rotating magnetic field produced by the rotor.

<u>The Induced Voltage in a 3-Phase Set of Coils</u>

If the stator now has 3 sets of different windings as such that the stator voltage induced due to the rotating magnetic field produced by the rotor will have a phase difference of 120º, the induced voltages at each phase will be as follows:

$$\begin{aligned} e_{aa'} &= N\phi\omega \sin \omega t \quad V \\ e_{bb'} &= N\phi\omega \sin(\omega t - 120^o) \quad V \\ e_{cc'} &= N\phi\omega \sin(\omega t - 240^o) \quad V \end{aligned}$$



The production of three-phase voltages from three coils spaced 120º apart

Therefore, a 3 phase set of currents flowing into the stator windings and hence generating a rotating magnetic field (earlier case), and at the same time, a rotating magnetic field produced by the rotor will be able to generate 3 phase voltages in a stator.

Referring to the induced voltage derived earlier, the maximum induced voltage is when sin has a value of 1, hence,

$$E_{\max} = N\phi\omega \quad , \quad \text{since } \omega = 2\pi f,$$
$$\therefore E_{\max} = 2\pi N\phi f$$

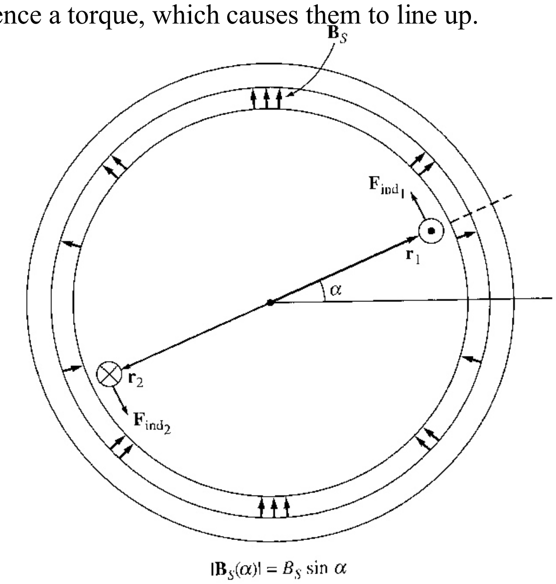
Therefore, the rms voltage at the 3 phase stator:

$$E_A = \sqrt{2}\pi N\phi f$$

Note: These are induced voltages at each phase, as for the line-line voltage values; it will depend upon how the stator windings are connected, whether as Y or Δ.

5. <u>Induced Torque in an AC Machines</u>

In ac machines under normal operating conditions, there are 2 magnetic fields present - a magnetic field from the rotor circuit and another magnetic field from the stator circuit. The interaction of these two magnetic fields produces the torque in the machine, just as 2 permanent magnets near each other will experience a torque, which causes them to line up.



A simplified ac machine with a sinusoidal stator flux distribution and a single coil of wire mounted in the rotor.

There will also be current flowing through the rotor windings (this will create another magnetic field originating from the wire), which will create force that can be found using the right hand rule. Its resultant direction may be found in the diagram above.

The stator flux density distribution in this machine is

$$B_S(\alpha)=B_S \sin \alpha$$

Where B_S is the magnitude of the peak flux density; $B_S(\alpha)$ is positive when the flux density vector points radially outward from the rotor surface to the stator surface.

How much torque is produced in the rotor of this simplified ac machine? This is done by analyzing the force and torque on each of the two conductors separately:

The induced force on conductor *l* is

$$\begin{aligned} F &= i\left(l \times B\right) \\ &= ilB_s \sin\alpha \end{aligned}$$

Hence torque at conductor 1:

$$\begin{aligned} \tau_{ind1} &= \left(r \times F\right) \\ &= rilB_s \sin\alpha \;\; (\textit{same direction as force}) \end{aligned}$$

The same may be found for conductor 2, hence the total torque induced:

$$\tau_{ind} = 2rilB_s \sin\alpha \;\; (\textit{same direction as force})$$

The current *i* flowing in the rotor coil produces a magnetic field of its own. The direction of the peak of this magnetic field is given by the right hand rule, and the magnitude of its magnetizing intensity H_R is directly proportional to the current flowing in the rotor, and $H_R = Ci$ where C is a constant of proportionality.

The angle between the peak of the stator flux density B_S and the peak of the rotor magnetizing intensity H_R is γ. Futhermore,

$$\begin{aligned} &\gamma = 180^o - \alpha \\ &\therefore \sin\gamma = \sin\left(180^o - \alpha\right) = \sin\alpha \end{aligned}$$

Therefore the torque equation may be represented in the following form:

$$\tau_{ind} = KH_rB_s \sin\alpha = KH_r \times B_s$$

Note that K is a constant value.
Since $B_R = \mu H_R$,

$$\tau_{ind} = kB_r \times B_s$$

The constant k is a value which will be dependent upon the permeability of the machine's material. Since the total magnetic field density will be the summation of the B_S and B_R, hence:

$$\tau_{ind} = kB_r \times \left(B_{net} - B_r\right) = kB_r \times B_{net}$$

If there is an angle δ between B_{net} and B_R,

$$\tau_{ind} = kB_rB_{net} \sin\delta$$

These 3 equations will be used to help develop a qualitative understanding of the torque in ac machines.

CHAPTER 5 – SYNCHRONOUS GENERATOR

Summary:

1. **Synchronous Generator Construction**

2. **The Speed of Rotation of a Synchronous Generator**

3. **The Internal Generated Voltage of a Synchronous Generator**

4. **The Equivalent Circuit of a Synchronous Generator**

5. **The Phasor Diagram of a Synchronous Generator**

6. **Power and Torque in Synchronous Generator**

7. **Measuring Synchronous Generator Model Parameters**

8. **The Synchronous Generator Operating Alone**
   - The Effect of Load Changes on a Synchronous Generator Operating Alone.

9. **Parallel operation of AC Generators**
   - The conditions required for paralleling
   - The general procedure for paralleling generators
   - Frequency-power and Voltage-Reactive Power characteristics of a synchronous generator.
   - Operation of generators in parallel with large power systems
   - Operation of generators in parallel with other generators of the same size.

10. **Synchronous Generator Ratings**
    - The Voltage, Speed and Frequency Ratings
    - Apparent Power and Power-Factor Ratings
    - Synchronous Generator Capability Curve

1. Synchronous Generator Construction

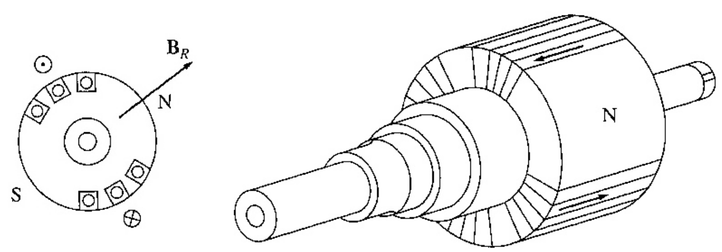
A DC current is applied to the rotor winding, which then produces a rotor magnetic field. The rotor is then turned by a prime mover (eg. Steam, water etc.) producing a rotating magnetic field. This rotating magnetic field induces a 3-phase set of voltages within the stator windings of the generator.

"Field windings" applies to the windings that produce the main magnetic field in a machine, and "armature windings" applies to the windings where the main voltage is induced. For synchronous machines, the field windings are on the rotor, so the terms "rotor windings" and "field windings" are used interchangeably.

Generally a synchronous generator must have at least 2 components:

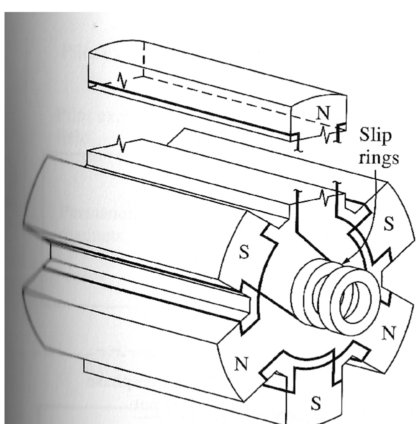
a) Rotor Windings or Field Windings
    a. Salient Pole
    b. Non Salient Pole
b) Stator Windings or Armature Windings

The rotor of a synchronous generator is a large electromagnet and the magnetic poles on the rotor can either be salient or non salient construction. Non-salient pole rotors are normally used for rotors with 2 or 4 poles rotor, while salient pole rotors are used for 4 or more poles rotor.

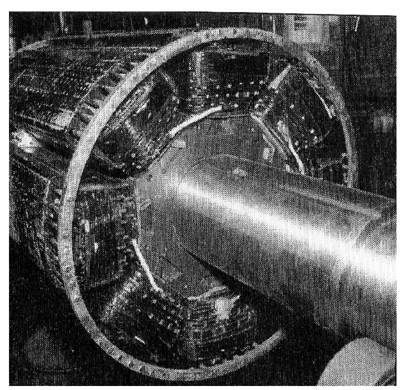


Side View

Non-salient rotor for a synchronous machine



(a)



Salient rotor

A dc current must be supplied to the field circuit on the rotor. Since the rotor is rotating, a special arrangement is required to get the dc power to its field windings. The common ways are:

a) supply the dc power from an external dc source to the rotor by means of slip rings and brushes.
b) Supply the dc power from a special dc power source mounted directly on the shaft of the synchronous generator.

Slip rings are metal rings completely encircling the shaft of a machine but insulated from it. One end of the dc rotor winding is tied to each of the 2 slip rings on the shaft of the synchronous machine, and a stationary brush rides on each slip ring.

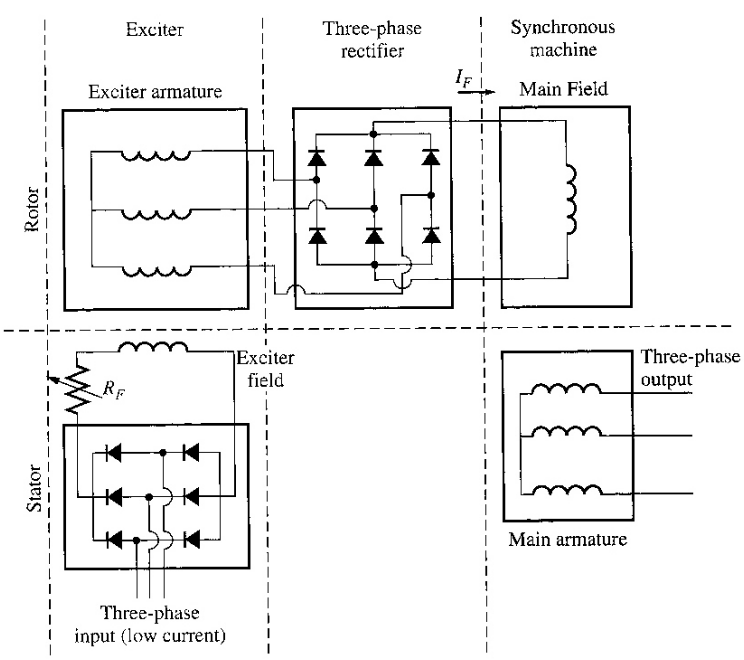
A "brush" is a block of graphitelike carbon compound that conducts electricity freely but has very low friction, hence it doesn't wear down the slip ring. If the positive end of a dc voltage source is connected to one brush and the negative end is connected to the other, then the same dc voltage will be applied to the field winding at all times regardless of the angular position or speed of the rotor.

Some problems with slip rings and brushes:

- They increase the amount of maintenance required on the machine, since the brushes must be checked for wear regularly.
- Brush voltage drop can be the cause of significant power losses on machines with larger field currents.

Small synchronous machines – use slip rings and brushes.
Larger machines – brushless exciters are used to supply the dc field current.

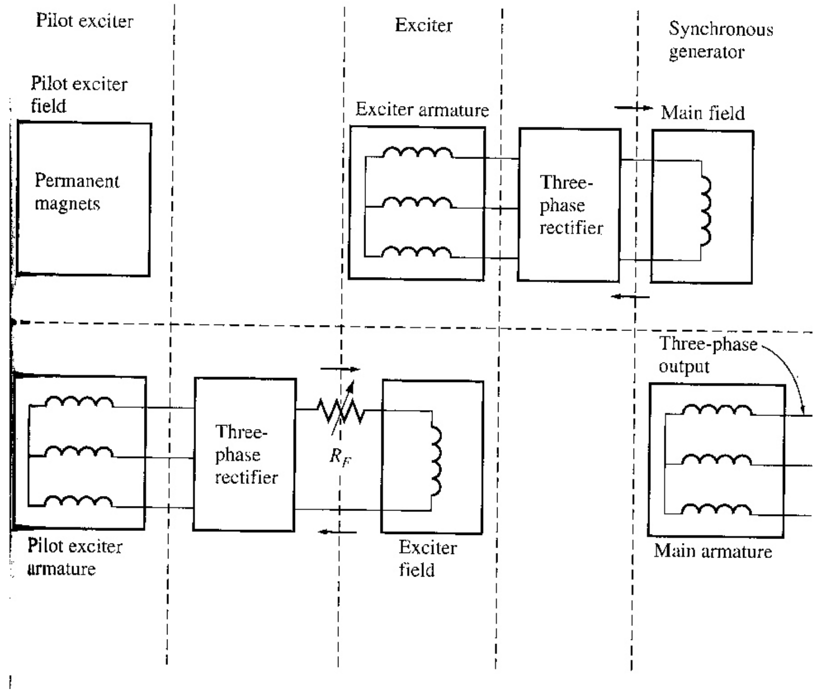
A brushless exciter is a small ac generator with its field circuit mounted on the stator and its armature circuit mounted on the rotor shaft. The 3-phase output of the exciter generator is rectified to direct current by a 3-phase rectifier circuit also mounted on the shaft of the generator, and is then fed to the main dc field circuit. By controlling the small dc field current of the exciter generator (located on the stator), we can adjust the field current on the main machine without slip rings and brushes. Since no mechanical contacts occur between the rotor and stator, a brushless exciter requires less maintenance.



A brushless exciter circuit : A small 3-phase current is rectified and used to supply the field circuit of the exciter, which is located on the stator. The output of the armature circuit of the exciter (on the rotor) is then rectified and used to supply the field current of the main machine.

To make the excitation of a generator completely independent of any external power sources, a small pilot exciter can be used.

A pilot exciter is a small ac generator with permanent magnets mounted on the rotor shaft and a 3-phase winding on the stator. It produces the power for the field circuit of the exciter, which in turn controls the field circuit of the main machine. If a pilot exciter is included on the generator shaft, then no external electric power is required.



A brushless excitation scheme that includes a pilot exciter. The permanent magnets of the pilot exciter produce the field current of the exciter, which in turn produces the field current of the main machine.

Even though machines with brushless exciters do not need slip rings and brushes, they still include the slip rings and brushes so that an auxiliary source of dc field current is available in emergencies.

2. The Speed of Rotation of a Synchronous Generator

Synchronous generators are by definition *synchronous*, meaning that the electrical frequency produced is locked in or synchronized with the mechanical rate of rotation of the generator. A synchronous generator's rotor consists of an electromagnet to which direct current is supplied. The rotor's magnetic field points in the direction the rotor is turned. Hence, the rate of rotation of the magnetic field in the machine is related to the stator electrical frequency by:

$$f_e = \frac{n_m P}{120}$$

3. The Internal Generated Voltage of a Synchronous Generator

Voltage induced is dependent upon flux and speed of rotation, hence from what we have learnt so far, the induced voltage can be found as follows:

$$E_A = \sqrt{2}\pi N_C \phi f$$

For simplicity, it may be simplified to as follows:

$$E_A = K\phi\omega$$

$$K = \frac{N_C P}{\sqrt{2}} \left(if\ \omega\ in\ electrical\ rads/s\right) \qquad K = \frac{N_C P}{2\sqrt{2}} \left(if\ \omega\ in\ mechanical\ rads/s\right)$$

4. <u>The Equivalent Circuit of a Synchronous Generator</u>

The voltage E_A is the internal generated voltage produced in one phase of a synchronous generator. If the machine is not connected to a load (no armature current flowing), the terminal voltage will be equivalent to the voltage induced at the stator coils. This is due to the fact that there are no current flow in the stator coils hence no losses. When there is a load connected to the generator, there will be differences between E_A and V_ϕ. These differences are due to:

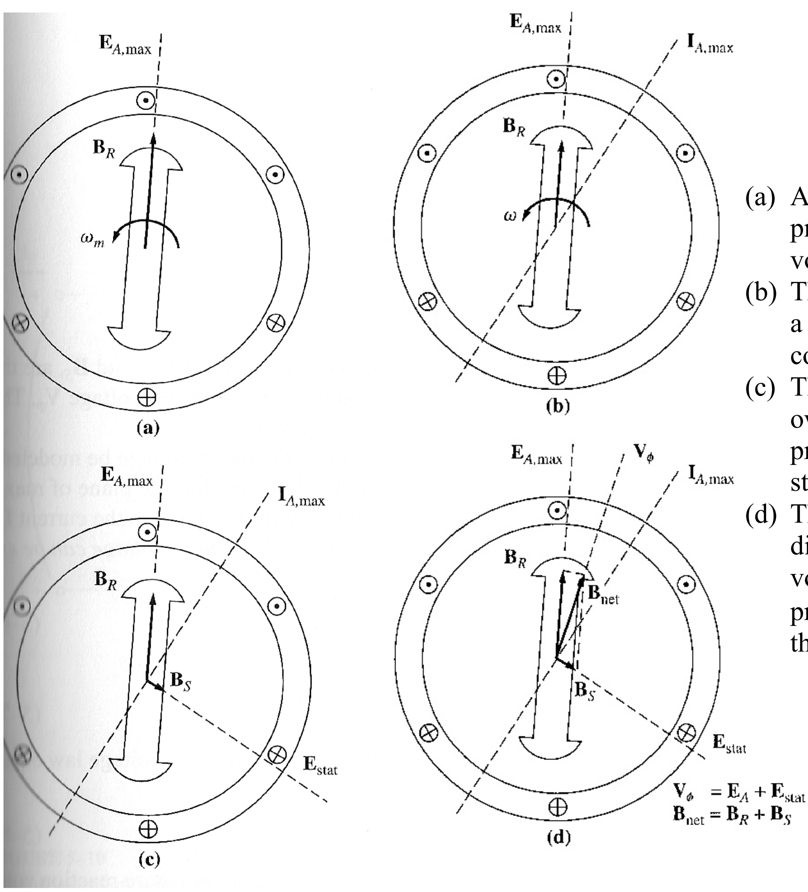
a) Distortion of the air gap magnetic field by the current flowing in the stator called **armature reaction**.
b) Self inductance of the armature coil
c) Resistance of the armature coils
d) The effect of salient pole rotor shapes.

We will explore factors a, b, and c and derive a machine model from them. The effect of salient pole rotor shape will be ignored, and all machines in this chapter are assumed to have nonsalient or cylindrical rotors.

Armature Reaction

When the rotor is spun, a voltage E_A is induced in the stator windings. If a load is attached to the terminals of the generator, a current flows. But a 3-phase stator current flow will produce a magnetic field of its own. This stator magnetic field will distorts the original rotor magnetic field, changing the resulting phase voltage. This effect is called armature reaction because the armature (stator) current affects the magnetic field, which produced it in the first place.

Refer to the diagrams below, showing a two-pole rotor spinning inside a 3-phase stator.



(a) A rotating magnetic field produces the internal generated voltage E_A.
(b) The resulting voltage produces a lagging current flow when connected to a lagging load.
(c) The stator current produces its own magnetic field B_S which produces its own E_{stat} in the stator windings.
(d) The field B_S adds to B_R distorting it into B_{net}. The voltage E_{stat} adds to E_A, producing V_ϕ at the output of the phase.

(a) There is no load connected to the stator. The rotor magnetic field B_R produces an internal generated voltage E_A whose peak coincides with direction of B_R. With no load, there is no armature current and E_A will be equal to the phase voltage V_ϕ.
(b) When a lagging load is connected, the peak current will occur at an angle behind the peak voltage.
(c) The current flowing in the stator windings produces a magnetic field of its own. This stator magnetic field B_S and its direction are given by the right-hand rule. The stator field produces a voltage of its own called E_{stat}.
(d) With 2 voltages and 2 magnetic fields present in the stator windings, the total voltage and the net magnetic field are:

$$V_\phi = E_A + E_{Stat}$$
$$B_{net} = B_R + B_S$$

How can the effects of armature reaction on the phase voltage be modeled?

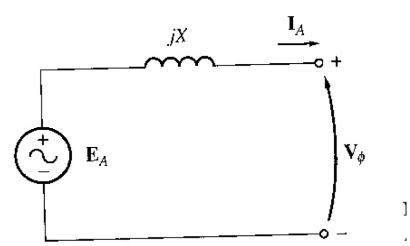
- The voltage E_{stat} lies at an angle of 90° behind the plane of I_A.
- The voltage E_{stat} is directly proportional to the current I_A.

If X is a constant of proportionality, then the armature reaction voltage can be expressed as:

$$E_{stat} = -jXI_A$$

Therefore:

$$V_\phi = E_A - jXI_A$$



Thus, the armature reaction voltage can be modeled as an inductor in series with the internal generated voltage.

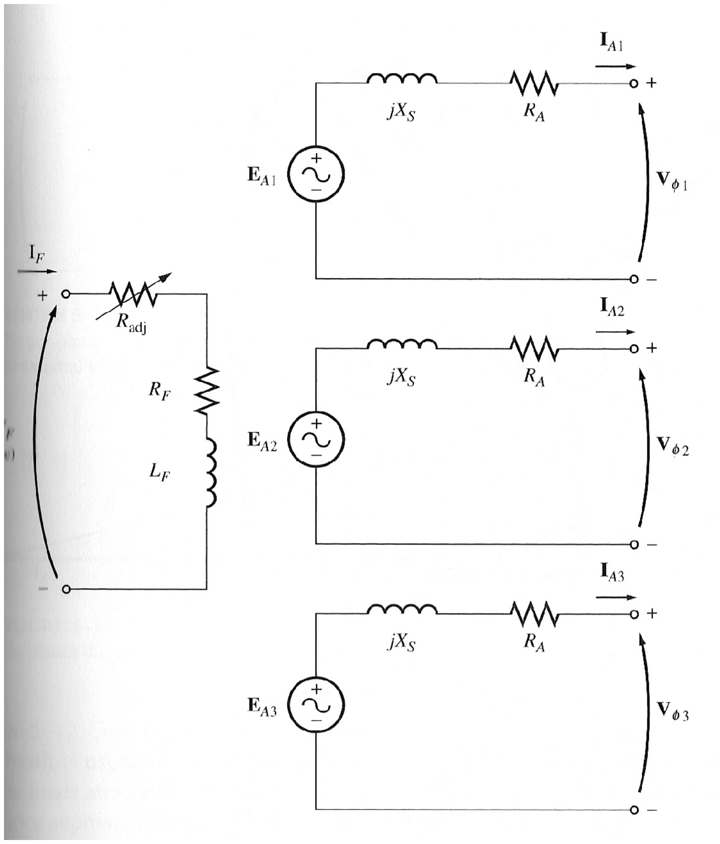
Self-inductance and Resistance of the Armature Coils

If the stator self-inductance is called L_A (reactance is X_A) while the stator resistance is called R_A, then the total difference between E_A and V_ϕ is:

$$V_\phi = E_A - jXI_A - jX_AI_A - R_AI_A$$
$$= E_A - jX_sI_A - R_AI_A$$

Where $X_S = X + X_A$

The full equivalent circuit is shown below:



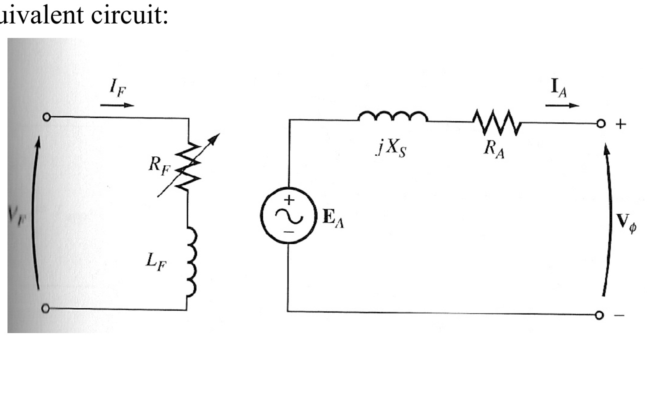
A dc power source is supplying the rotor field circuit, whis is modeled by the coil's inductance and resistance in series. In series with R_F is an adjustable resistor R_{adj} which controls the flow of the field current. The rest of the equivalent circuit consists of the models for each phase. Each phase has an internal generated voltage with a series inductance X_S (consisting of the sum of the armature reactance and the coil's self-inductance) and a series resistance R_A.

If the 3 phases are connected in Y or Δ, the terminal voltage may be found as follows:

$$V_T = \sqrt{3}V_\phi \quad (for\ Y\ connection)$$
$$V_T = V_\phi \quad (for\ \Delta\ connection)$$

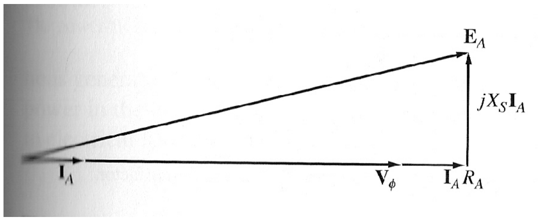
Ideally, the terminal voltage for all 3 phases should be identical since we assume that the load connected is balanced. If it is not balanced, a more in-depth technique is required.

The per-phase equivalent circuit:

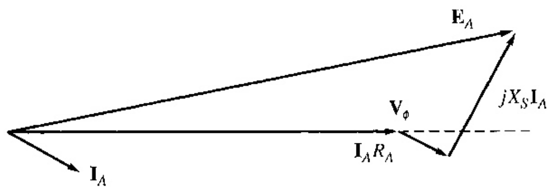


5. Phasor Diagram of a Synchronous Generator

Similar concept as applied in Chapter 2 (Transformers). The phasor diagrams are as follows:

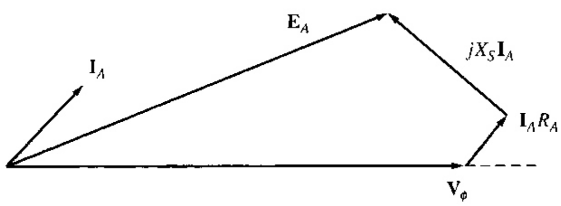


Unity power factor



(a)

Lagging power factor



(b)

Leading power factor

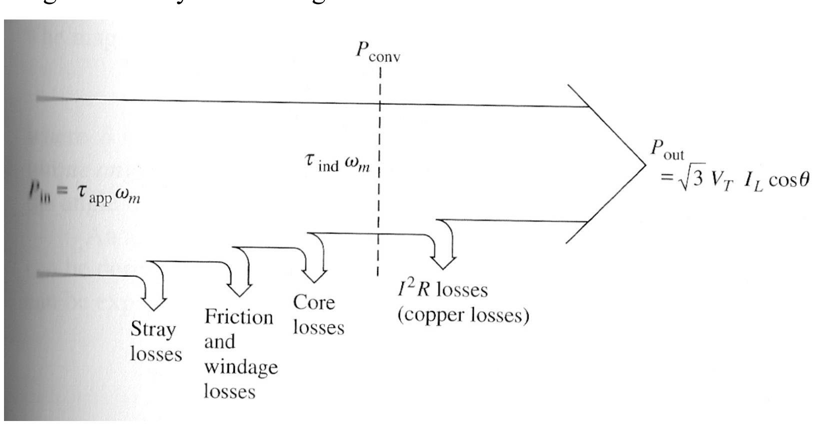
For a given phase voltage and armature current, a larger internal voltage E_A is needed for lagging loads than for leading loads. Thus, a larger field current is needed to get the same terminal voltage because $E_A = k\phi\omega$ because ω must be kept constant to keep constant frequency.

Alternatively, for a given field current and magnitude of load current, the terminal voltage is lower for lagging loads and higher for leading loads.

6. Power and Torque in Synchronous Generators

A generator converts mechanical energy into electrical energy, hence the input power will be a mechanical prime mover, e.g. diesel engine, steam turbine, water turbine or anything similar. Regardless of the type of prime mover, the rotor velocity must remain constant to maintain a stable system frequency.

The power-flow diagram for a synchronous generator is shown:



Input: $P_{in} = \tau_{app}\omega_m$

Losses: Stray losses, friction and windage losses, core loss

Converted power: $P_{conv} = \tau_{ind}\omega_m = 3E_A I_A \cos\gamma$

Where γ is the angle between E_A and I_A.

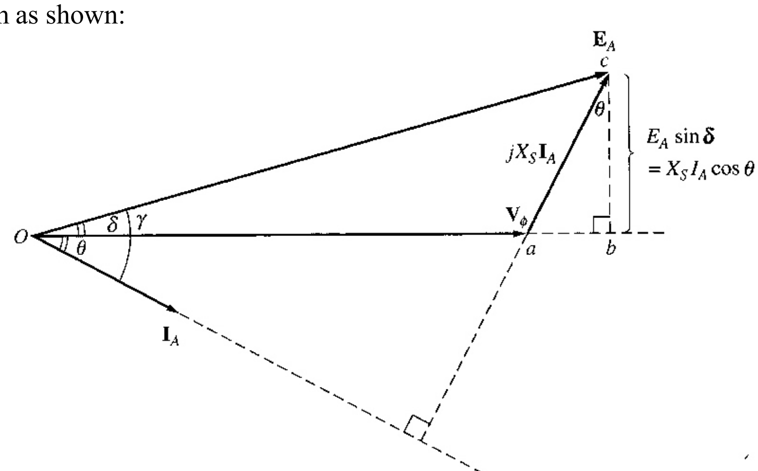
Losses: Copper losses

Output:

$$P_{out} = \sqrt{3}V_T I_L \cos\theta \qquad or \qquad P_{out} = 3V_\phi I_A \cos\theta$$

$$Q_{out} = \sqrt{3}V_T I_L \sin\theta \qquad or \qquad P_{out} = 3V_\phi I_A \sin\theta$$

Simplifying the phasor diagram, an assumption may be made whereby the armature resistance R_A is considered to be negligible and assuming that load connected to it is lagging in nature. This gives a phasor diagram as shown:



Based upon the simplified phasor diagram:

$$I_A \cos\theta = \frac{E_A \sin\delta}{X_s}$$

Which gives another form of output power expression (since R_A assumed to be zero):

$$P = \frac{3V_\phi E_A \sin\delta}{X_s}$$

From the above equation, it can be seen that power is dependent upon:

- The angle between V_ϕ and E_A which is δ.
- δ is known as the torque angle of the machine.
- maximum torque may be found when $\sin\delta$ is 1 which gives the maximum power (a.k.a. static stability limit) to be:

$$P_{\max} = \frac{3V_\phi E_A}{X_s}$$

The basic torque equation:

$$\tau_{ind} = kB_R \times B_s = kB_R \times B_{net} = kB_R B_{net} \sin\delta$$

An alternative expression can be derived from the power expression since $P_{out}=P_{conv}$ when R_A assumed to be zero. Because $P_{conv}=\tau_{ind}\omega_m$, the induced voltage is:

$$\tau_{ind} = \frac{3V_\phi E_A \sin\delta}{\omega_m X_s}$$

7. Measuring Synchronous Generator Model Parameters

There are basically 3 types of relationship which needs to be found for a synchronous generator:

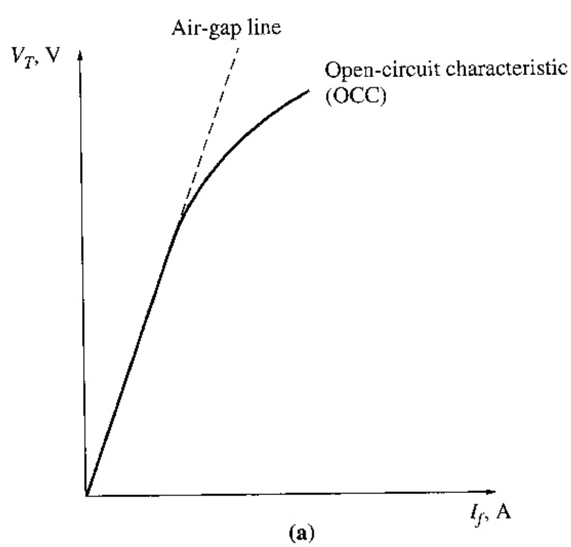
a) Field current and flux relationship (and thus between the field current and E_A)
b) Synchronous reactance
c) Armature resistance

Open Circuit test

Steps:

1) Generator is rotated at the rated speed.
2) No load is connected at the terminals.
3) Field current is increased from 0 to maximum.
4) Record values of the terminal voltage and field current value.

With the terminals open, $I_A=0$, so $E_A = V_\phi$. It is thus possible to construct a plot of E_A or V_T vs I_F graph. This plot is called open-circuit characteristic (OCC) of a generator. With this characteristic, it is possible to find the internal generated voltage of the generator for any given field current.



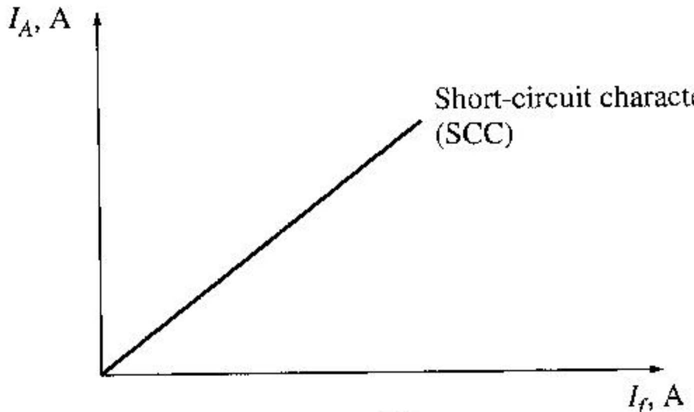
Open-circuit characteristic (OCC) of a generator

At first the curve is almost perfectly linear, until some saturation is observed at high field currents. The unsaturated iron in the frame of the synchronous machine has a reluctance several thousand times lower than the air-gap reluctance, so at first almost all the mmf is across the air-gap, and the resulting flux increase is linear. When the iron finally saturates, the reluctance of the iron increases dramatically, and the flux increases much more slowly with an increase in mmf. The linear portion of an OCC is called the air-gap line of the characteristic.

Short circuit test

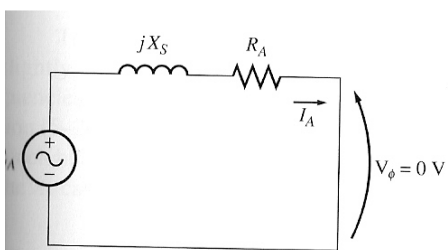
Steps:
1) Generator is rotated at rated speed.
2) Adjust field current to 0.
3) Short circuit the terminals.
4) Measure armature current or line current as the field current is increased.

Notes: During the short circuit analysis, the net magnetic field is very small, hence the core is not saturated, hence the reason why the relationship is linear.



(b)

SCC is essentially a straight line. To understand why this characteristic is a straight line, look at the equivalent circuit below when the terminals are short circuited.

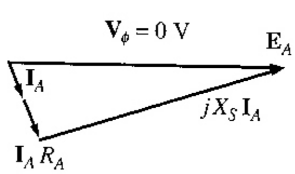


(a)

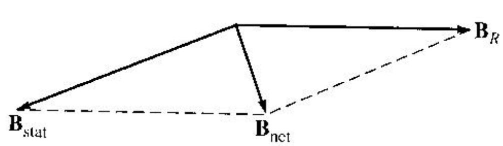
When the terminals are short circuited, the armature current I_A is: $$I_A = \frac{E_A}{R_A + jX_S}$$

And its magnitude is: $$I_A = \frac{E_A}{\sqrt{R_A^2 + X_S^2}}$$

The resulting phasor diagram and the corresponding magnetic fields are shown below:



(b)



(c)

Since B_S almost cancels B_R, the net magnetic field B_{net} is very small (corresponding to internal resistive and inductive drops only). Since the net magnetic field is small, the machine is unsaturated and the SCC is linear.

From both tests, here we can find the internal machine impedance (E_A from OCC, I_A fom SCC):

$$Z_S = \sqrt{R_A^2 + X_S^2} = \frac{E_A}{I_A}$$

Since $X_s >> R_A$, the equation reduces to:

$$X_s \approx \frac{E_A}{I_A} = \frac{V_{\phi oc}}{I_A}$$

Therefore we may be able to find the synchronous reactances.

Therefore, an approximate method for determining the synchronous reactance X_S at a given field current is:

1. Get the internal generated voltage E_A from the OCC at that field current.
2. Get the short circuited current flow $I_{A,SC}$ at that field current from the SCC.
3. Find X_S by applying the equation above.

Problem with this method:

E_A is taken from the OCC whereby the core would be *partially saturated* for large field currents while I_A is taken from the SCC where the core is *not saturated* at all field currents. Therefore E_A value taken during the OCC may not be the same E_A value in the SCC test. Hence the value of X_S is only an approximate.

Hence to gain better accuracy, the test should be done at low field currents which looks at the linear region of the OCC test.

To find out on the resistive element of the machine, it can simply be found by applying a DC voltage to the machine terminals with the rotor stationary. Value obtained in this test (R_A) may increase the X_S accuracy.

Short Circuit Ratio

Definition:

Ratio of the field current required for the rated voltage at open circuit to the field current required for rated armature current at short circuit.

Example 5-1

A 200kVA, 480V, 50Hz, Y-connected synchronous generator with a rated field current of 5A was tested, and the following data were taken:

1. $V_{T,OC}$ at the rated I_F was measured to be 540V
2. $I_{L,SC}$ at the rated I_F was found to be 300A.
3. When a dc voltage of 10V was applied to two of the terminals, a current of 25A was measured.

Find the values of the armature resistance and the approximate synchronous reactance in ohms that would be used in the generator model at the rated conditions.

8. The synchronous generator operating alone

The behaviour of a synchronous generator under load varies greatly depending on the power factor of the load and on whether the generator is operating alone or in parallel with other synchronous generator. The next discussion, we shall disregard R_A and rotor flux is assumed to be constant unless it is stated that the field current is changed. Also, the speed of the generator will be assumed constant, and all terminal characteristics are drawn assuming constant speed.

The Effect of Load Changes on a Synchronous Generator Operating Alone

Assume a generator is connected to a load.

Load increase:
An increase of load is an increase in real and reactive power drawn from the generator. Such a load increase increases the load current drawn from the generator.

Assumptions:
- Field resistor has not been changed, field current is kept constant, hence flux is constant.
- Generator rotor speed is maintained constant.
- Therefore E_A is constant.

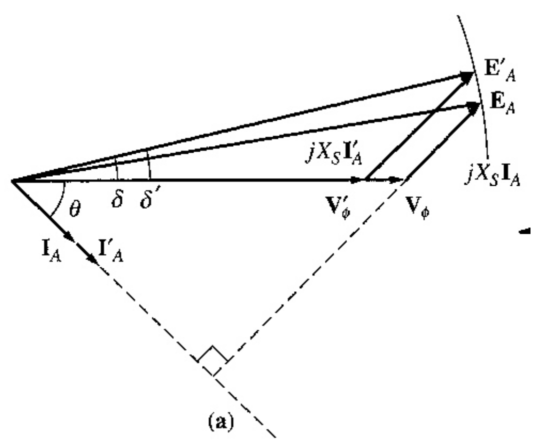
If E_A *is constant, what actually varies with a changing load??*

Initially lagging load:
- Load is increased with the lagging power factor maintained.
- Magnitude of I_A will increase but will maintain the same angle with reference to V_ϕ. (due to power factor is maintained lagging)
- $X_S I_A$ will also increase and will maintain the same angle. Since

$$E_A = V_\phi + jX_s I_A$$

j $X_S I_A$ must strecth between V_ϕ at an angle of 0° and E_A, which is constrained to be of the same magnitude as before the load increase.

- Note that E_A has to remain constant (refer to the assumption stated earlier)
- Hence the only element which would change to compensate would be V_ϕ. This change may be seen in the phasor diagram.



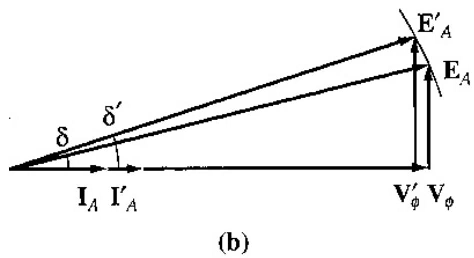
The effect of an increase in generator loads at constant power factor upon its terminal voltage – lagging power factor.

Initially unity load:

- Load is increased with the unity power factor maintained.
- Magnitude of I_A will increase but will maintain the same angle with reference to V_ϕ. (due to power factor is maintained unity)
- $X_S I_A$ will also increase and will maintain the same angle. Since

$$E_A = V_\phi + jX_s I_A$$

- Note that E_A has to remain constant (refer to the assumption stated earlier)
- Hence the only element which would change to compensate would be V_ϕ. This change may be seen in the phasor diagram.



The effect of an increase in generator loads at constant power factor upon its terminal voltage – unity power factor.

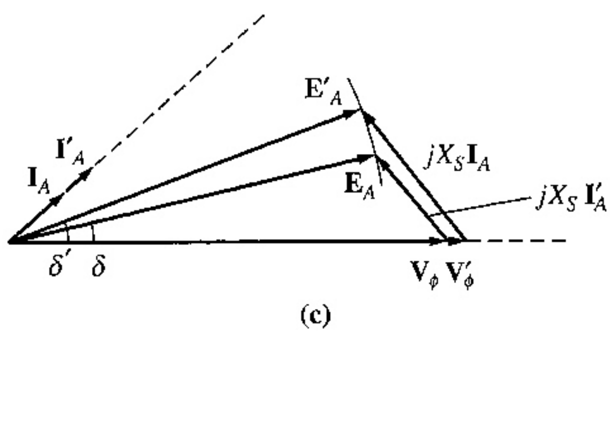
- Changes in V_ϕ would be decreasing but it would be less significant as compared to when the load is lagging.

Initially leading load:

- Load is increased with the leading power factor maintained.
- Magnitude of I_a will increase but will maintain the same angle with reference to V_ϕ. (due to power factor is maintained leading)
- $X_s I_a$ will also increase and will maintain the same angle. Since

$$E_A = V_\phi + jX_s I_A$$

- Note that E_a has to remain constant (refer to the assumption stated earlier)
- Hence the only element which would change to compensate would be V_ϕ. This change may be seen in the phasor diagram.



The effect of an increase in generator loads at constant power factor upon its terminal voltage – leading power factor.

An alternative way to explain this is via the **voltage regulation formulae**.

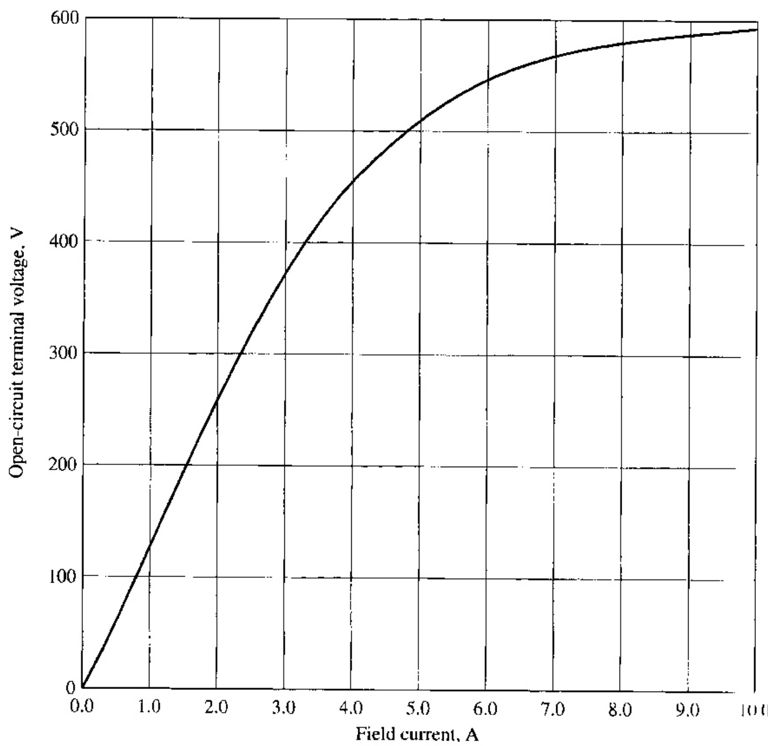
- For lagging loads, VR would be very positive.
- For leading loads, VR would be very negative.
- For unity loads, VR would be positive.

However, in practical it is best to keep the **output voltage of a generator to be constant**, hence **E_A has to be controlled** which can be done by **controlling the field current I_F. Varying I_F will vary the flux in the core which then will vary E_A accordingly (refer OCC).**

How must a generator's field current be adjusted to keep V_T constant as the load changes?

Example 5-2

A 480V, 60Hz, Δ-connected, 4-pole synchronous generator has the OCC as shown below. This generator has a synchronous reactance of 0.1Ω and an armature resistance of 0.015 Ω. At full load, the machine supplies 1200A at 0.8 PF lagging. Under full-load conditions, the friction and windage losses are 40kW, and the core losses aree 30kW. Ignore any field circuit losses.



(a) hat is the speed of rotation of this generator?
(b) How much field current must be supplied to the generator to make the terminalvoltage 480V at no load?
(c) If the generator is now connected to a load and the load draws 1200A at 0.8 PF lagging, how much field current will be required to keep the terminal voltage equal to 480V?
(d) How much power is the gen now supplying? How much power is supplied to the generator by the prime mover? What is this machine's overall efficiency?
(e) If the generator's load were suddenly disconnected from the line, what would happen to its terminal voltage?
(f) Finally, suppose that the generator is connected to a load drawing 1200A at 0.8 PF leading. How much field current would be required to keep V_T at 480V?

Example 5-3

A 480V, 50Hz, Y-connected, 6-pole synchronous generator has a per-phase synchronous reactance of 1Ω. Its full-load armature current is 60A at 0.8PF lagging. This generator has friction and windage losses of 1.5kW and core losses of 1 kW at 60Hz at full load. Since the armature resistance is being ignored, assume that the I^2R losses are negligible. The field current has been adjusted so that the terminal voltage is 480V at no load.

(a) What is the speed of rotation of this generator?
(b) What is the terminal voltage of this generator if the following are true?
    1. It is loaded with the rated current at 0.8 PF lagging.
    2. It is loaded with the rated current at 1.0 PF.
    3. It is loaded with the rated current at 0.8 PF leading.
(c) What is the efficieny of this generator (ignoring the unknown electrical losses) when it is operating at the rated current and 0.8 PF lagging?
(d) How much shaft torque must applied by the prime mover at full load? How large is the induced countertorque?
(e) What is the voltage regulation of this generator at 0.8 PF lagging? At 1.0 PF? At 0.8 PF leading?

9. Parallel Operation of AC Generators

Reasons for operating in parallel:

a) Handling larger loads.
b) Maintenance can be done without power disruption.
c) Increasing system reliability.
d) Increased efficiency.

Conditions required for Paralleling

The figure below shows a synchronous generator G1 supplying power to a load, with another generator G2 about to be paralleled with G1 by closing switch S1. What conditions must be met before the switch can be closed and the 2 generators connected?

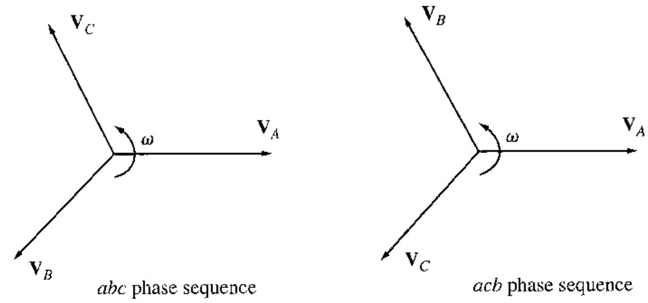
If the switch is closed arbitrarily at some moment, the generators are liable to be severely damaged, and the load may lose power.

If the voltages are not exactly the same in each conductor being tied together, there will be a very large current flow when the switch is closed. To avoid this problem, each of the three phases must have exactly the same voltage magnitude and phase angle as the conductor to which it is connected.

Thus, paralleling 2 or more generators must be done carefully as to avoid generator or other system component damage. Conditions are as follows:

a) RMS line voltages must be equal.

b) The generators to be paralleled must have the same phase sequence. If the phase sequence is different (as shown here), then even though one pair of voltages (the *a* phase) is in phase, the other 2 pairs of voltages are 120° out of phase. If the generators were connected in this manner, there would be no problem with phase *a*, but huge currents would flow in phases *b* and *c*, damaging both machines.



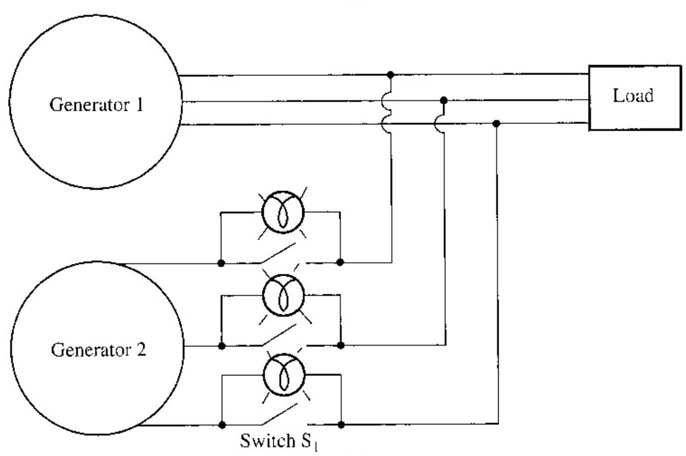
(a)

c) Generator output phase angles must be the same.

d) The **oncoming generator** (the new generator) must have a slightly higher operating frequency as compared to the system frequency. This is done so that the phase angles of the incoming machine will change slowly with respect to the phase angles of the running system.

General Procedure for Paralleling Generators

Suppose that generator G2 is to be connected to the running system as shown below:



(b)

1. Using Voltmeters, the **field current** of the oncoming generator should be adjusted until its **terminal voltage is equal to the line voltage** of the running system.

2. Check and verify **phase sequence** to be identical to the system phase sequence. There are 2 methods to do this:
   i. Alternately connect a small induction motor to the terminals of each of the 2 generators. If the motor rotates in the same direction each time, then the phase sequence is the same for both generators. If the motor rotates in opposite directions, then the phase sequences differ, and 2 of the conductors on the incoming generator must be reversed.

   ii. Another way is using the 3 light bulb method, where the bulbs are stretched across the open terminals of the switch connecting the generator to the system (as shown in the figure above). As the phase changes between the 2 systems, the light bulbs first get bright (large phase difference) and then get dim (small phase difference). If all 3 bulbs get bright and dark together, then the systems have the same phase sequence. If the bulbs brighten in succession, then the systems have the opposite phase sequence, and one of the sequences must be reversed.

iii. Using a Synchroscope – a meter that measures the difference in phase angles (it does not check phase sequences only phase angles).

3. Check and verify **generator frequency** to be slightly higher than the system frequency. This is done by watching a frequency meter until the frequencies are close and then by observing changes in phase between the systems.

4. Once the frequencies are nearly equal, the voltages in the 2 systems will change phase with respect to each other very slowly. The phase changes are observed, and when the phase angles are equal, the switch connecting the 2 systems is shut.

Frequency-Power and Voltage-Reactive Power Characteristics of a Synchronous Generator

All generators are driven by a prime mover, which is the generator's source of mechanical power. All prime movers tend to behave in a similar fashion – as the power drawn from them increases, the speed at which they turn decreases. The decrease in speed is in general non linear, but some form of governor mechanism is usually included to make the decrease in speed linear with an increase in power demand.

Whatever governor mechanism is present on a prime mover, it will always be adjusted to provide a slight drooping characteristic with increasing load. The speed droop (SD) of a prime mover is defined as:

$$SD = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\%$$

Where n_{nl} is the no-load prime mover speed and n_{fl} is the full-load prime mover speed.

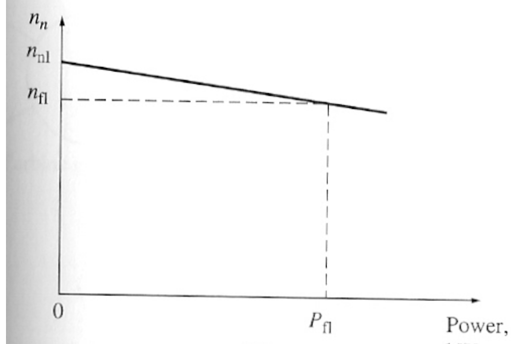
Typical values of SD are 2% – 4%. Most governors have some type of set point adjustment to allow the no-load speed of the turbine to be varied. A typical speed vs. power plot is as shown below.

Since mechanical speed is related to the electrical frequency and electrical frequency is related with the output power, hence we will obtain the following equation:

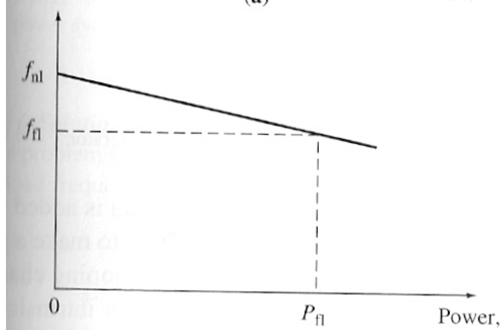
$$P = s_p \left(f_{nl} - f_{sys} \right)$$

Where
P = output power
f_{nl} = no-load frequency of the generator
f_{sys} = operating frequency of system
s_P = slope of curve in kW/Hz or MW/Hz

If we look in terms of reactive power output and its relation to the terminal voltage we shall see a similar shape of curve as shown in the frequency power curve.



(a)



(b)

In conclusion, for a **single** generator:

a) For any given real power, the governor set points control the generator operating frequency
b) For any given reactive power, the field current controls the generator's terminal voltage.
c) Real and reactive power supplied will be the amount demanded by the load attached to the generator – the P and Q supplied cannot be controlled by the generator's controls.

Example 5-5

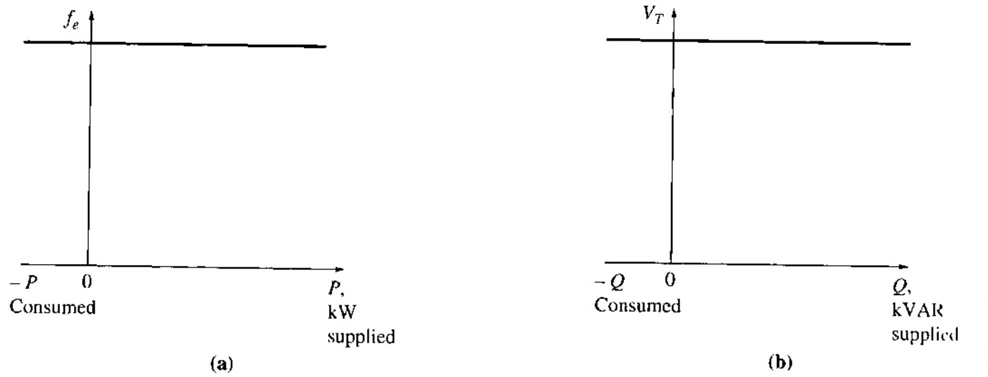
Figure below shows a generator supplying a load. A second load is to be connected in parallel with the first one. The generator has a no-load frequency of 61.0 Hz and a slope s_p of 1 MW/Hz. Load 1 consumes a real power of 1000kW at 0.8 PF lagging, while load 2 consumes a real power of 800kW at 0.707 PF lagging.

(a) Before the switch is closed, what is the operating frequency of the system?
(b) After load 2 is connected, what is the operating frequency of the system?
(c) After load 2 is connected, what action could an operator take to restore the system frequency to 60Hz?

Operation of Generators in Parallel with Large Power Systems

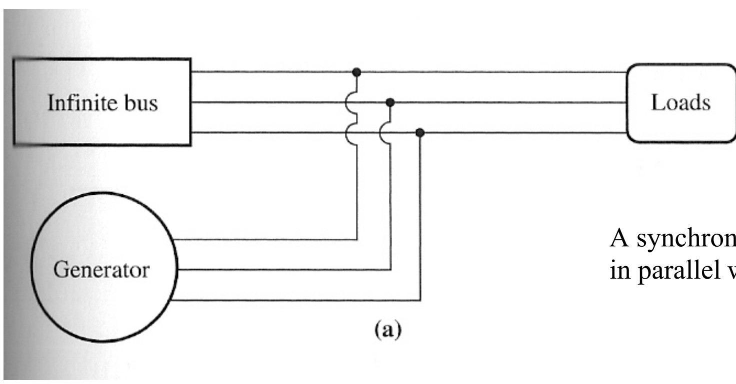
Changes in one generator in large power systems may not have any effect on the system.

A large power system may be represented as an **infinite bus** system. An infinite bus is a power system so large that its voltage and frequency do not vary regardless of how much real and reactive power is drawn from or supplied to it. The power-frequency characteristic and the reactive power-voltage characteristic are shown below:

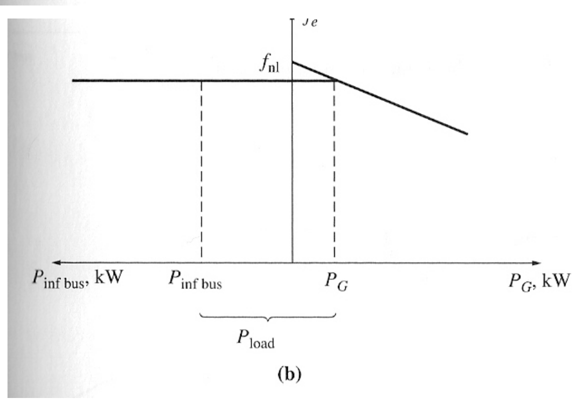


Now consider a generator connected to an infinite bus system feeding into a load. We shall consider the action or changes done to the generator and its effect to the system. Assume that the generator's prime mover has a governor mechanism, but that the field is controlled manually by a resistor.

When a generator is connected in parallel with another generator or a large system, the frequency and terminal voltage of all the machines must be the same, since their output conductors are tied together. Thus, their real power-frequency and reactive power-voltage characteristics can be plotted back to back, with a common vertical axis. Such a sketch is called a *house diagram*, as shown below:

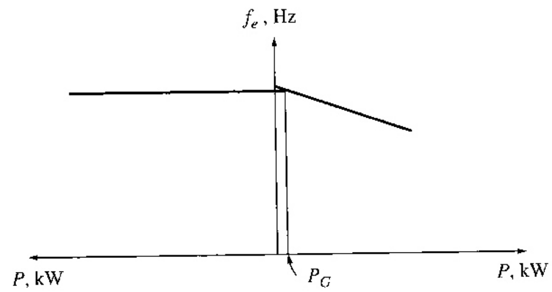


A synchronous generator operating in parallel with an infinite bus

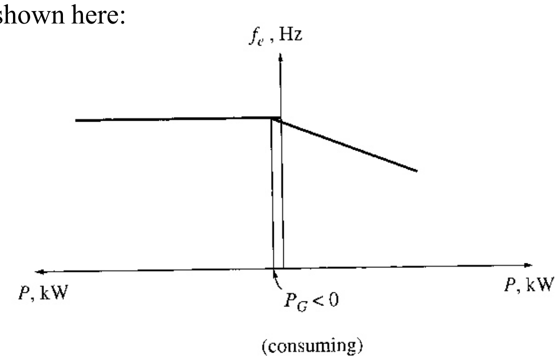


The frequency-power diagram (house diagram) for a synchronous generator in parallel with an infinite bus.

Assume that the generator has just been paralleled with the infinite bus according to the procedure described previously. Thus, the generator will be "floating" on the line, supplying a small amount of real power and little or no reactive power. This is shown here:



Suppose the generator had been paralleled to the line but instead of being at a slightly higher frequency than the running system, it was at a slightly lower frequency. In this case, when paralleling is completed, the resulting situation is as shown here:

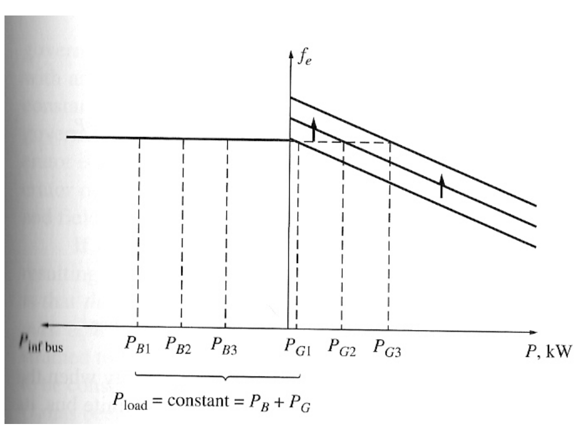


Notice that here the no-load frequency of the generator is less than the system's frequency. At this frequency, the power supplied by the generator is actually negative. In other words, when the generator's no-load frequency is less than the system's operating frequency, the generator actually consumes electric power and runs as a motor. It is to ensure that a generator comes on line supplying power instead of consuming in that the oncoming machine's frequency is adjusted higher than the running system's frequency.

Assume that the generator is already connected, what effects of governor control and field current control has to the generator?

Governor Control Effects:

In theory, if the governor set points is increased, the no load frequency will also increase (the droop graph will shift up). Since in an infinite bus system frequency does not change, the overall effect is to increase the generator output power (another way to explain that it would look as if the generator is loaded up further). Hence the output current will increase.



The effect of increasing the governor's set point on the house diagram

The effect of increasing the governor's set point on the phasor diagram

Notice that in the phasor diagram that $E_A \sin \delta$ (which is proportional to the power supplied as long as V_T is constant) has increased, while the magnitude of E_A ($=K\phi\omega$) remains constant, since both the field current I_F and the speed of rotation ω is unchanged. As the governor set points are further increased the no-load frequency increases and the power supplied by the generator increases. As the power output increases, E_A remains at constant magnitude while $E_A \sin \delta$ is further increased.

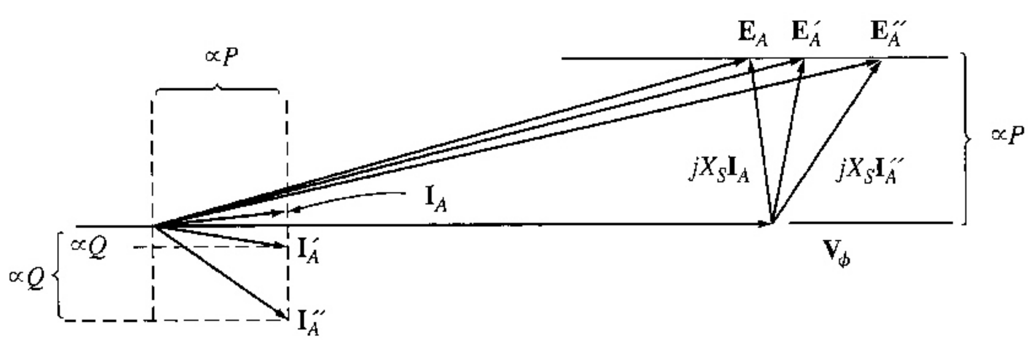
If the governor is set as such that it exceeds the load requirement, the excess power will flow back to the infinite bus system. The infinite bus, by definition, can supply or consume any amount of power without a change in frequency, so the extra power is consumed.

Field Current Control Effects:

Increasing the governor set point will increase power but will cause the generator to absorb some reactive power. The question is now, how do we supply reactive power Q into the system instead of absorbing it? This can be done by adjusting the field current of the generator.

Constraints: Power into the generator must remain constant when I_F is changed so that power out of the generator must also remain constant. The power into a generator is given by the equation $P_{in}=\tau_{ind}\omega_m$. Now, the prime mover of a synchronous generator has a fixed-torque speed characteristic for any given governor setting. This curve changes only when the governor set points are changed. Since the generator is tied to an infinite bus, its speed cannot change. If the generator's speed does not change and the governor set points have not been changed, the power supplied by the generator must remain constant.

If the power supplied is constant as the field current is changed, then the distances proportional to the power in the phasor diagram ($I_A \cos \theta$ and $E_A \sin \delta$) cannot change. When the field current is increased, the flux ϕ increases, and therefore E_A ($=K\ \phi\uparrow\ \omega$) increases. If E_A increases, but $E_A \sin \delta$ must remain constant, then the phasor E_A must "slide" along the line of constant power, as shown below.



The effect of increasing the generator's field current on the phasor diagram of the machine.

Since V_ϕ is constant, the angle of jX_SI_A changes as shown, and therefore the angle and magnitude of I_A change. Notice that as a result the distance proportional to Q ($I_A\sin\theta$) increases.

In other words, increasing the field current in a synchronous generator operating in parallel with an infinite bus increases the reactive power output of the generator.

Hence, for a generator operating in parallel with an infinite bus:

a) **Frequency and terminal voltage of generator is controlled by the connected system.**
b) **Changes in Governor set points will control real power to be supplied.**
c) **Changes in Field Current will control the amount of reactive power to be supplied.**

Note that these effects are only applicable for generators in a large system only.

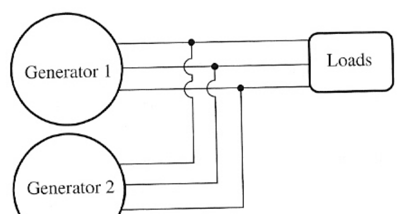
Operation of Generators in Parallel with Other Generators of the Same Size

When a single generator operated alone, the real and reactive powers supplied by the generators are fixed, constrained to be equal to the power demanded by the load, and the frequency and terminal voltage were varied by the governor set points and the field current.

When a generator is operating in parallel with an infinite bus, the frequency and terminal voltage were constrained to be constant by the infinite bys, and the real and reactive powers were varied by the governor set points and the field current.

What happens when a synchronous generator is connected in parallel not with an infinite bus, but rather with another generator of the same size?
What will be the effect of changing governor set points and field currents?

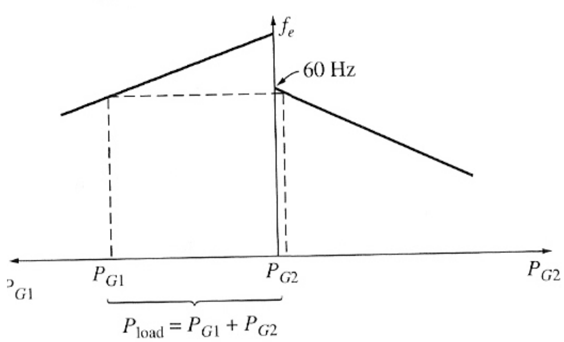
The system is as shown here:



(a)

In this system, the basic constraint is that the sum of the real and reactive powers supplied by the two generators must equal the P and Q demanded by the load. The system frequency is not constrained to be constant, and neither is the power of a given generator constrained to be constant.

The power-frequency diagram for such a system immediately after G_2 has been paralleled to the line is shown below:



(b)

The house diagram at the moment G_2 is paralleled with the system

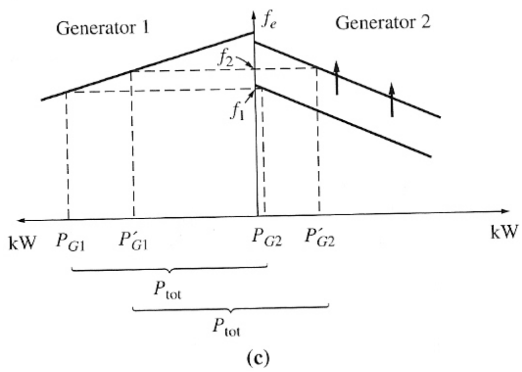
The total power P_{tot} (which is equal to P_{load}) and reactive power respectively are given by:

$P_{tot} = P_{load} = P_{G1} + P_{G2}$

$Q_{tot} = Q_{load} = Q_{G1} + Q_{G2}$

What happens if the governor set points of G_2 are increased?

As a result, the power-frequency curve of G_2 shifts upward as shown here:



The effect of increasing G_2's governor set points on the operation of the system.

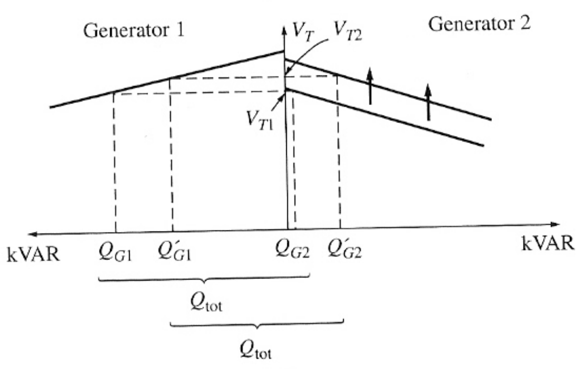
The total power supplied to the load must not change. At the original frequency f_1, the power supplied by G_1 and G_2 will now be larger than the load demand, so the system cannot continue to operate at the same frequency as before. In fact, there is only one frequency at which the sum of the powers out of the two generators is equal to P_{load}. That frequency f_2 is higher than the original system operating frequency. At that frequency, G_2 supplies more power than before, and G_1 supplies less power than before.

Thus, when 2 generators are operating together, an increase in governor set points on one of them

1. increases the system frequency.
2. increases the power supplied by that generator, while reducing the power supplied by the other one.

What happens if the field current of G_2 is increased?

The resulting behaviour is analogous to the real-power situation as shown below:



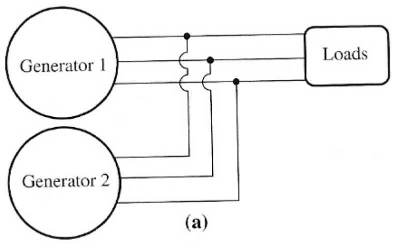
The effect of increasing G_2's field current on the operating of the system.

When 2 generators are operating together and the field current of G_2 is increased,

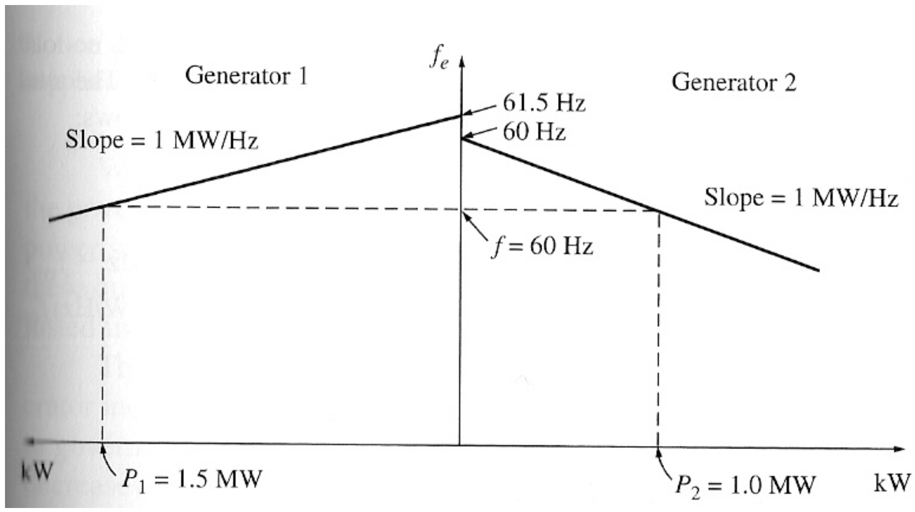
1. The system terminal voltage is increased.
2. The reactive power Q supplied by that generator is increased, while the reactive power supplied by the other generator is decreased.

If the slopes and no-load frequencies of the generator's speed droop (frequency-power) curves are known, then the powers supplied by each generator and the resulting system frequency can be determined quantitatively. Example 5-6 shows how this can be done.

Example 5-6



G_1 has a no-load frequency of 61.5 Hz and a slope s_{P1} of 1MW/Hz. G_2 has a no-load frequency of 61Hz and a slope s_{P1} of 1MW/Hz. The 2 generators are supplying a real load totalling 2.5 MW at 0.8 PF lagging. The resulting system power-frequency or house diagram is shown below.

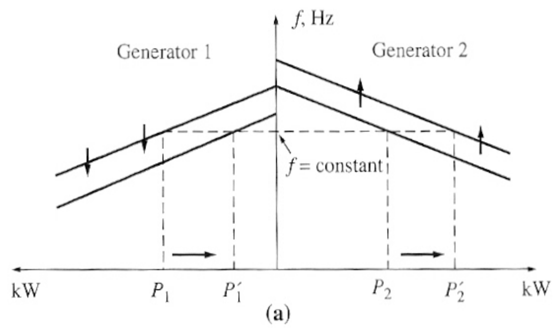


(a) At what frequency is this system operating, and how much power is supplied by each of the 2 generators?
(b) Suppose an additional 1-MW load were attached to this power system. What would the new system frequency be, and how much power would G_1 and G_2 supply now?
(c) With the system in the configuration described in part b, what will the system frequency and generator powers be of the governor set points on G_2 are increased by 0.5 Hz?

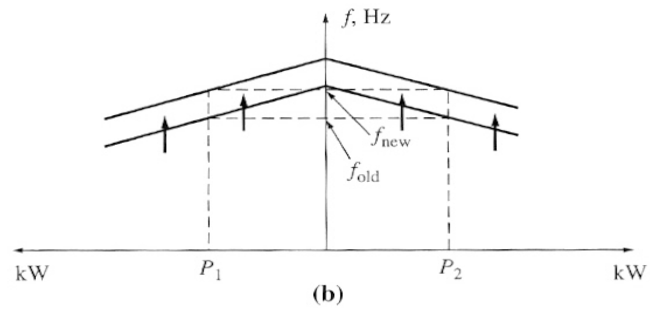
When 2 generators of similar size are operating in parallel, a change in the governor set points of one of them changes both the system freq and the power sharing between them.

How can the power sharing of the power system be adjusted independently of the system frequency, and vice versa?

An increase in governor set points on one generator increases that machine's power and increases the system frequency. A decrease in governor set points on the other generator decreases that machine's power and decreases the system frequency. Therefore, to adjust power sharing without changing the system frequency, increase the governor set points of one generator and simultaneously decrease the governor set points of the other generator. (and same goes when adjusting the system frequency). This is shown below:

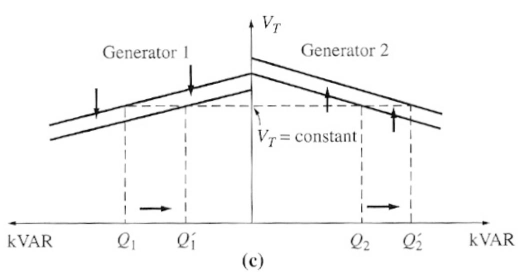


Shifting power sharing without affecting system frequency

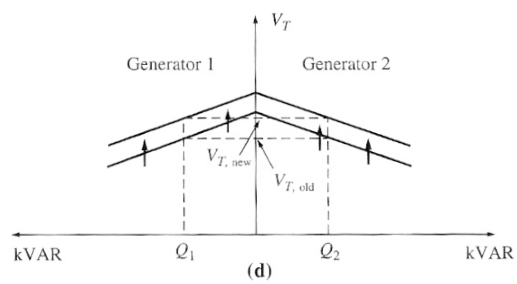


Shifting system frequency without affecting power sharing

Reactive power and terminal voltage adjustment work in an analogous fashion. To shift the reactive power sharing without changing V_T, simultaneously increase the field current on one generator and decrease the field current on the other. (and same goes when adjusting the terminal voltage). This is shown below:

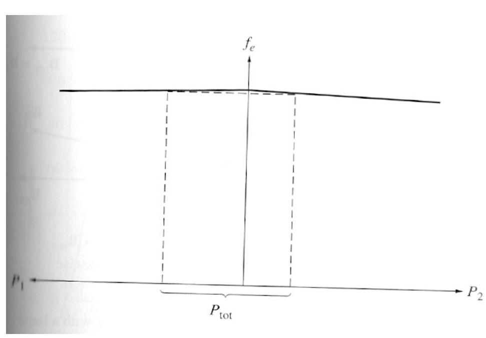


Shifting reactive power sharing without affecting terminal voltage



Shifting terminal voltage without affecting reactive power sharing

It is very important that any synchronous generator intended to operate in parallel with other machines have a drooping frequency-power characteristic. If two generators have flat or nearly flat characteristics, then the power sharing between them can vary widely with only the tiniest changes in no-load speed. This problem is illustrated below:



Notice that even very tiny changes in f_{nl} in one of the generators would cause wild shifts in power sharing. To ensure good control of power sharing between generators, they should have speed droops in the range of 2-5%.

10. Synchronous Generator Ratings

The Voltage, Speed and Frequency Ratings

Frequency Ratings: Rated frequency will depend upon the system at which the generator is connected.

Voltage Ratings: Generated voltage is dependent upon flux, speed of rotation and mechanical constants. However, there is a ceiling limit of flux level since it is dependent upon the generator material. Hence voltage ratings may give a rough idea on its maximum flux level possible and also maximum voltage to before the winding insulation breaks down.

Apparent Power and Power Factor Ratings

Constraints for electrical machines generally dependent upon mechanical strength (mechanical torque on the shaft of the machine) and also its winding insulation limits (heating of its windings). For a generator, there are 2 different windings that has to be protected which are:

a) Armature winding
b) Field Winding

Hence the maximum armature current flow can be found from the maximum apparent power, S:

$$S = 3V_\phi I_A$$

If the rated voltage is known, we may find the maximum I_A allowed.

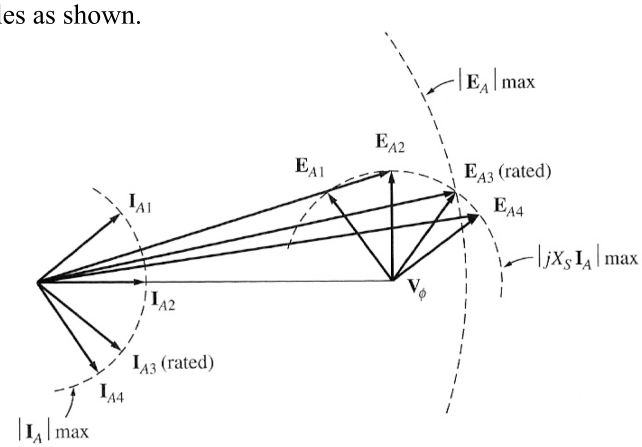
The heating effect of the stator copper losses is given by:

$$P_{SCL} = 3\ I_A^2\ R_A$$

The field copper losses:

$$P_{RCL} = I_F^2 R_F$$

Maximum field current will set the maximum E_A permissible. And since we can find the maximum field current and the maximum E_A possible, we may be able to determine the lowest PF changes possible for the generator to operate at rated apparent power. Figure below shows the phasor diagram of a synchronous generator with the rated voltage and armature current. The current can assume many different angles as shown.

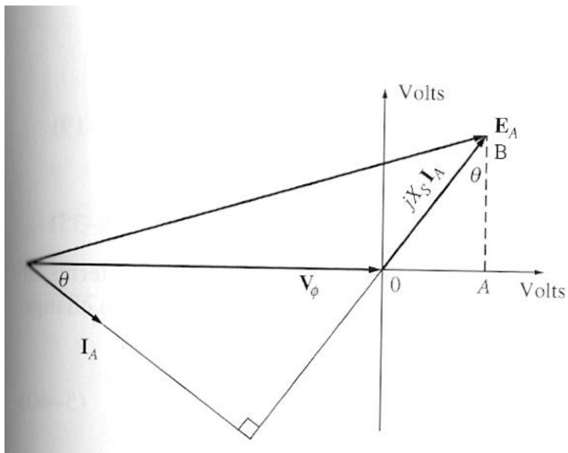


The internal generated voltage E_A is the sum of $V\phi$ and jX_SI_A. Notice that for some possible current angles the required E_A exceeds $E_{A,max}$. If the generator were operated at the rated armature current and these power factors, the field winding would burn up.
The angle of I_A that requires the max possible E_A while $V\phi$ remains at the rated value gives the rated power factor of the generator. It is possible to operate the generator at a lower (more lagging) power factor than the rated value, but only by cutting back on the kVA supplied by the generator.

Synchronous Generator Capability Curves.

Based upon these limits, there is a need to plot the capability of the synchronous generator. This is so that it can be shown graphically the limits of the generator.

A capability diagram is a plot of complex power S=P+jQ. The capability curve can be derived back from the voltage phasor of the synchronous generator. Assume that a voltage phasor as shown, operating at lagging power factor and its rated value:



(a)

Note that the capability curve of the must represent power limits of the generator, hence there is a need to convert the **voltage phasor** into **power phasor**.

The powers are given by:

$$P = 3\ V\phi\ I_A \cos\theta$$
$$Q = 3\ V\phi\ I_A \sin\theta$$
$$S = 3\ V\phi\ I_A$$

Thus,

The conversion factor to change the scale of the axes from V to VA is $3\ V\phi/X_S$.

On the voltage axes, the origin of the phasor diagram is at -Vϕ on the horizontal axis, so the origin on the power diagram is at:

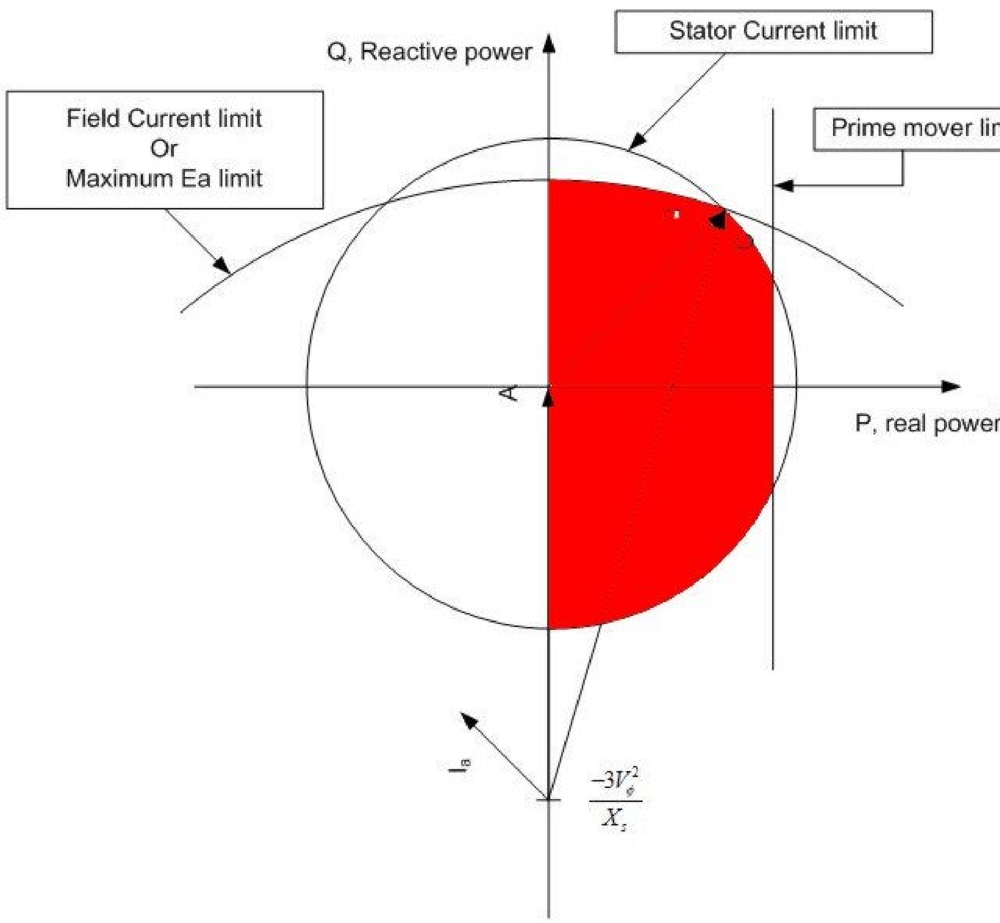
$$Q = \frac{3V_\phi}{X_S}\left(-V_\phi\right) = -\frac{3V_\phi}{X_S}$$

The field current is proportional to the machine's flux, and the flux is proportional to $E_A = K\phi\omega$. The length corresponding to E_A on the power diagram is:

$$D_E = \frac{3E_A V_\phi}{X_S}$$

The armature current I_A is proportional to $X_S I_A$, and the length corresponding to $X_S I_A$ on the power diagram is $3V\phi I_A$.

The final capability curve is shown below:



It is a plot of P vs Q. Lines of constant armature current I_A appear as lines of constant $S = 3V\phi I_A$, which are concentric circles around the origin. Lines of constant field current correspond to lines of constant E_A, which are shown as circles of magnitude $3E_A V\phi/X_S$ centered on the point

$$Q = -\frac{3V_\phi^2}{X_S}$$

The armature current limit appears as the circle corresponding to the rated I_A or rated KVA, and the field current limit appears as a circle corresponding to the rated I_F or E_A. Any point that lies within both circles is a safe operating point for the generator.

<u>Example 5-8</u>

A 480V, 50 Hz, Y-connected, 6-pole, synchronous generator is rated at 50kVA at 0.8 PF lagging. It has a synchronous reactance of 1 ohm per phase. Assume that this generator is connected to a steam turbine capable of supplying up to 45kW. The friction and windage losses are 1.5 kW, and the core losses are 1.0 kW.

(a) Sketch the capability curve for this generator, including the prime mover power limit.
(b) Can this generator supply a line current of 56A at 0.7 PF lagging? Why or why not?
(c) What is the max amount of reactive power this generator can produce?
(d) If the generator supplies 30kW of real power, what is the maximum amount of reactive power that can be simultaneously supplied?

Chapter 6: Synchronous Motor

In general, a synchronous motor is very similar to a synchronous generator with a difference of function only.

Steady State Operations

A synchronous motor are usually applied to instances where the load would require a constant speed. Hence for a synchronous motor, its torque speed characteristic is constant speed as the induced torque increases. Hence SR = 0%.

Since,

$$\tau_{ind} = kB_R B_{net} \sin\delta$$

$$\tau_{ind} = \frac{3V_\phi E_A \sin\delta}{\omega_m X_s}$$

Maximum torque (pullout torque) is achieved when sin δ = 1. If load exceeds the pullout torque, the rotor will slow down. Due to the interaction between the stator and rotor magnetic field, there would be a torque surge produced as such there would be a loss of synchronism which is known as **slipping poles**.

Also based upon the above equation, maximum induced torque can be achieved by increasing E_a hence increasing the field current.

Effect of load changes

Assumption:

> *A synchronous generator operating with a load connected to it. The field current setting are unchanged.*

Varying load would in fact slow the machine down a bit hence increasing the torque angle. Due to an increase to the torque angle, more torque is induced hence spinning the synchronous machine to synchronous speed again.

The overall effect is that the synchronous motor phasor diagram would have a bigger torque angle δ. In terms of the term E_a, since I_f is set not to change, hence the magnitude of E_a should not change as shown in the phasor diagram (fig.6-6). Since the angle of d changes, the armature current magnitude and angle would also change to compensate to the increase of power as shown in the phasor diagram (fig. 6-6).

Effect of field current changes on a synchronous motor

Assumption:

> *The synchronous generator is rotating at synchronous speed with a load connected to it. The load remains unchanged.*

As the field current is increased, E_a should increase. Unfortunately, there are constraints set to the machine as such that the power requirement is unchanged. Therefore since P is has to remain constant, it imposes a limit at which I_a and jX_sI_a as such that E_a tends to slide across a horizontal limit as shown in figure 6-8. I_a will react to the changes in E_a as such that its angle changes from a leading power factor to a lagging power factor or vice versa.

This gives a possibility to utilise the synchronous motor as a power factor correction tool since varying magnetic field would change the motor from leading to lagging or vice versa.

This characteristic can also be represented in the V curves as shown in figure 6-10.

Synchronous motor as a power factor correction

Varying the field current would change to amount of reactive power injected or absorbed by the motor. Hence if a synchronous motor is incorporated nearby a load which require reactive power, the synchronous motor may be operated to inject reactive power hence maintaining stability and lowering high current flow in the transmission line.

Starting Synchronous Motors

Problem with starting a synchronous motor is the initial production of torque which would vary as the stator magnetic field sweeps the rotor. As a result, the motor will vibrate and could overheat (refer to figure 6-16 for diagram explanations).

There are 3 different starting methods available:

a) Reduced speed of stator magnetic field – the aim is to reduce it slow enough as such that the stator will have time to follow the stator magnetic field.
b) External prime mover to accelerate the synchronous motor.
c) Damper windings or amortisseur windings.

Stator magnetic field speed reduction

The idea is to let the stator magnetic field to rotate slow enough as such that the rotor has time to lock on to the stator magnetic field. This method used to be impractical due to problems in reducing stator magnetic field.

Now, due to power electronics technology, frequency reduction is possible hence makes it a more viable solution.

Using a prime mover

This is a very straightforward method.

Motor Starting using Amortisseur windings

This is the most popular way to start an induction motor. Amortisseur windings are a special kind of windings which is shorted at each ends. Its concept is near similar to an induction motor hence in depth explanation can be obtained in the text book (page 345-348).

The final effect of this starting method is that the rotor will spin at near synchronous speed. Note that the rotor will never reach synchronous speed unless during that time, the field windings are switched on hence will enable the rotor to lock on to the stator magnetic field.

Effect of Amortisseur windings

The advantage of this starting method is that it acts as a damper as such that during transient cases at which the system frequency would vary significantly (varying frequency would affect the synchronous speed) hence the amortisseur windings may act as a dampening effect to slow down a fast machines and to speed up slow machines,

CHAPTER 7 – INDUCTION MOTOR

Summary:

1. **Induction Motor Construction**

2. **Basic Induction Motor Concepts**
   - The Development of Induced Torque in an Induction Motor.
   - The Concept of Rotor Slip.
   - The Electrical Frequency on the Rotor.

3. **The Equivalent Circuit of an Induction Motor.**
   - The Transformer Model of an induction Motor.
   - The Rotor Circuit Model.
   - The Final Equivalent Circuit.

4. **Powers and Torque in Induction Motor.**
   - Losses and Power-Flow diagram
   - Power and Torque in an Induction Motor.
   - Separating the Rotor Copper Losses and the Power Converted in an Induction Motor's Equivalent Circuit.

5. **Induction Motor Torque-Speed Characteristics**
   - Induced Torque from a Physical Standpoint.
   - The Derivation of the Induction Motor Induced-Torque Equation.
   - Comments on the Induction Motor Torque Speed Curve.
   - Maximum (Pullout) Torque in an Induction Motor.

6. **Variations in Induction Motor Toque-Speed Characteristics**
   - Control of Motor Characteristics by Cage Rotor Design.
   - Deep-Bar and Double-Cage rotor design.
   - Induction Motor Design Classes.

7. **Starting Induction Motors**

8. **Speed Control of Induction Motor**
   - Induction Motor Speed Control by Pole Changing.
   - Speed Control by Changing the Line Frequency.
   - Speed Control by Changing the Line Voltage.
   - Speed Control by Changing the Rotor Resistance.

9. **Determining Circuit Model Parameters**
   - The No-Load Test
   - The DC Test
   - The Locked-Rotor Test

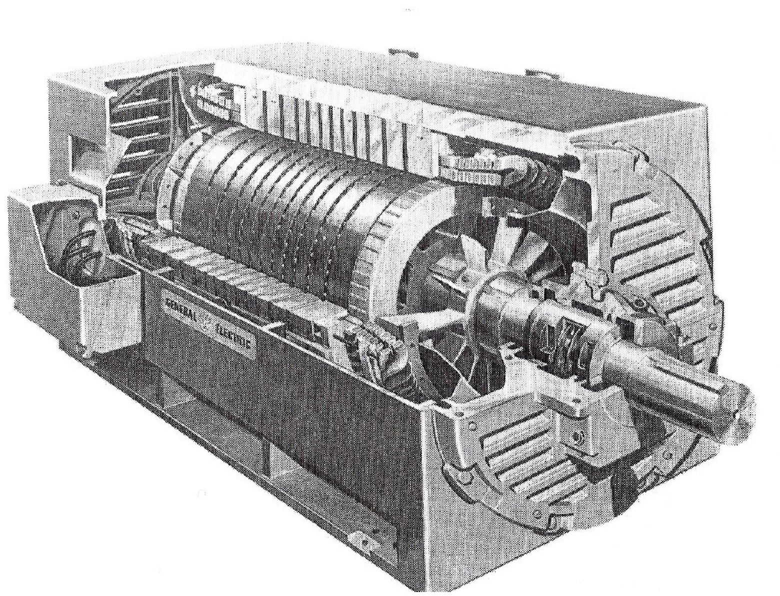
Induction machine – the rotor voltage that produces the rotor current and the rotor magnetic field is induced in the rotor windings rather than being physically connected by wires. No dc field current is required to run the machine.

1. Induction Motor Construction

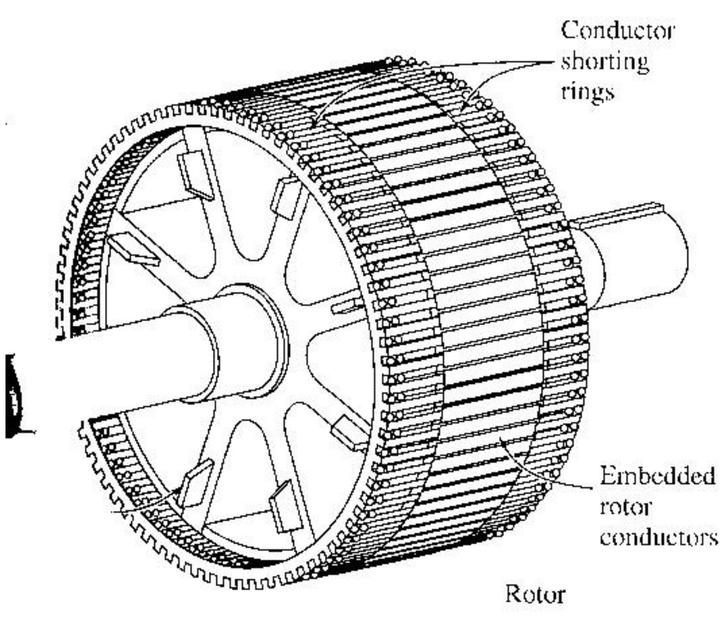
There are basically 2 types of rotor construction:

a) **Squirrel Cage -** no windings and no slip rings
b) **Wound rotor -** It has 3 phase windings, usually Y connected, and the winding ends are connected via slip rings.

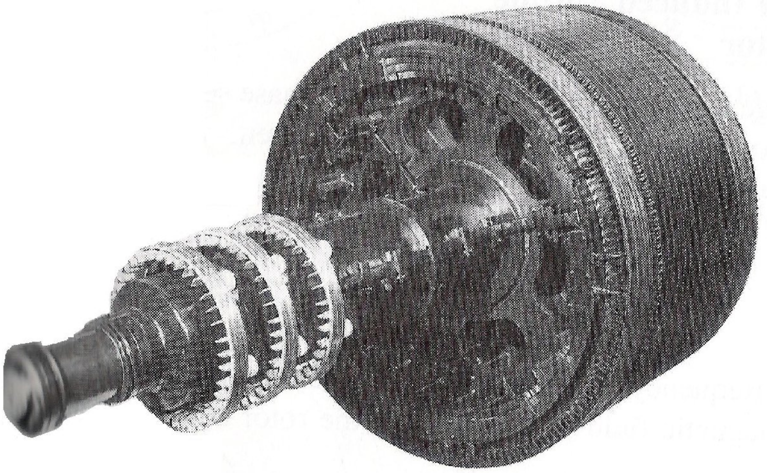
Wound rotor are known to be more expensive due to its maintenance cost to upkeep the slip rings, carbon brushes and also rotor windings.



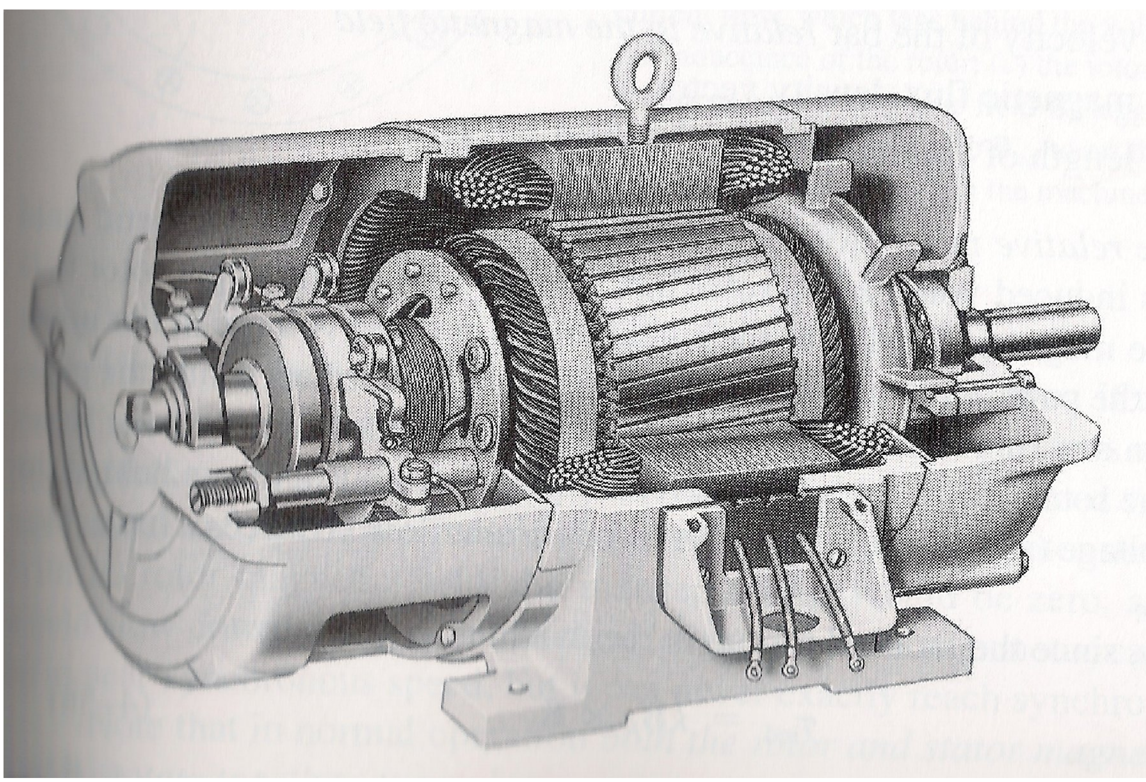
Cutaway diagram of a typical large cage rotor induction motor



Sketch of cage rotor



Typical wound rotor for induction motors.



Cutaway diagram of a wound rotor induction motor.

2. Basic Induction Motor Concepts

The Development of Induced Torque in an Induction Motor

When current flows in the stator, it will produce a magnetic field in stator as such that $\mathbf{B_s}$ (stator magnetic field) will rotate at a speed:

$$n_{sync} = \frac{120 f_e}{P}$$

Where f_e is the system frequency in hertz and P is the number of poles in the machine. This rotating magnetic field $\mathbf{B_s}$ passes over the rotor bars and induces a voltage in them. The voltage induced in the rotor is given by:

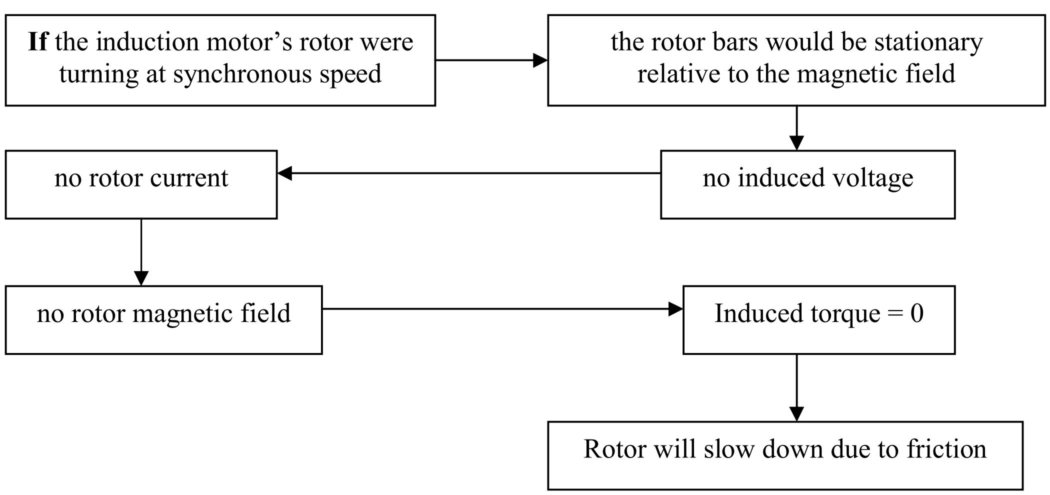
$$e_{ind} = (\mathbf{v} \times \mathbf{B})\, \boldsymbol{l}$$

Hence there will be rotor current flow which would be lagging due to the fact that the rotor has an inductive element. And this rotor current will produce a magnetic field at the rotor, $\mathbf{B_r}$. Hence the interaction between both magnetic field would give torque:

$$\tau_{ind} = kB_R \times B_S$$

The torque induced would generate acceleration to the rotor, hence the rotor will spin.

However, there is a finite upper limit to the motor's speed.



Conclusion : An induction motor can thus speed up to near synchronous speed but it can never reach synchronous speed.

The Concept of Rotor Slip

The induced voltage at the rotor bar is dependent upon the relative speed between the stator magnetic field and the rotor. This can be easily termed as **slip speed**:

$$n_{slip} = n_{sync} - n_m$$

Where n_{slip} = slip speed of the machine
n_{sync} = speed of the magnetic field.
n_m = mechanical shaft speed of the motor.

Apart from that we can describe this relative motion by using the concept of slip:

$$Slip,\ s = \frac{n_{slip}}{n_{sync}} \times 100\% = \frac{n_{sync} - n_m}{n_{sync}} \times 100\%$$

Slip may also be described in terms of angular velocity, ω.

$$s = \frac{\omega_{sync} - \omega_m}{\omega_{sync}} x100\%$$

Using the ratio of slip, we may also determine the rotor speed:

$$n_m = (1 - s)\, n_{sync} \qquad or \qquad \omega_m = (1 - s)\, \omega_{sync}$$

The Electrical Frequency on the Rotor

An induction motor is **similar to a rotating transformer where the primary is similar to the stator and the secondary would be a rotor.** But unlike a transformer, the secondary frequency may not be the same as in the primary.

If the rotor is locked (cannot move), the rotor would have the same frequency as the stator (refer to transformer concept). Another way to look at it is to see that when the rotor is locked, rotor speed drops to zero, hence by default, slip is 1. But as the rotor starts to rotate, the rotor frequency would reduce, and when the rotor turns at synchronous speed, the frequency on the rotor will be zero.

Why?
Since

$$S = \frac{n_{sync} - n_m}{n_{sync}}$$

And rotor frequency may be expressed as:

$$f_r = sf_e$$

Hence combing both equations would give:

$$f_r = \frac{n_{sync} - n_m}{n_{sync}} f_e$$

And since $n_{sync}=120f_e / P$,

$$f_r = \frac{P}{120}\left(n_{sync} - n_m\right)$$

Which shows that the relative difference between synchronous speed and the rotor speed will determine the rotor frequency.

Example 7.1

A 208V, 10hp, 4 pole, 60Hz, Y-connected induction motor has a full-load slip of 5%.

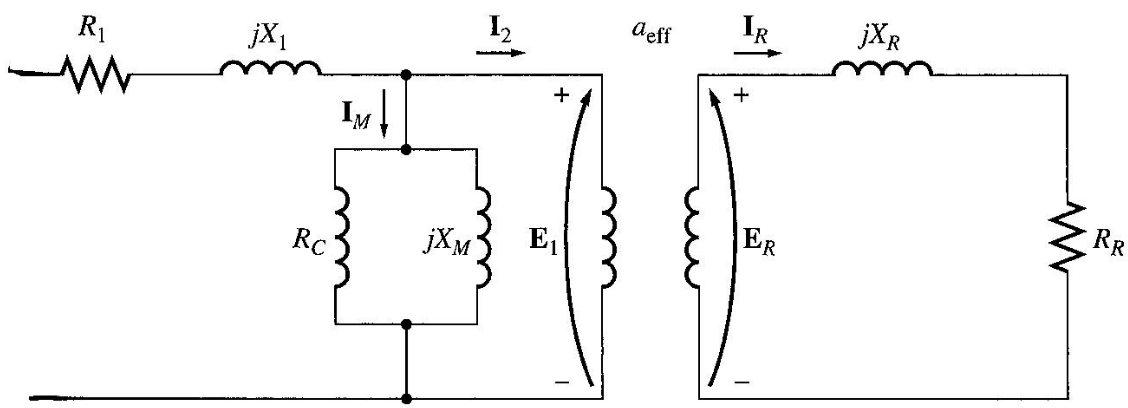
(a) What is the synchronous speed of this motor?
(b) What is the rotor speed of this motor at the rated load?
(c) What is the rotor frequency of this motor at the rated load?
(d) What is the shaft torque of this motor at the rated load?

3. The Equivalent Circuit of an Induction Motor

An induction motor relies for its operation on the induction of voltages and currents in its rotor circuit from the stator circuit (transformer action). This induction is essentially a transformer operation, hence the equivalent circuit of an induction motor is similar to the equivalent circuit of a transformer.

The Transformer Model of an Induction Motor

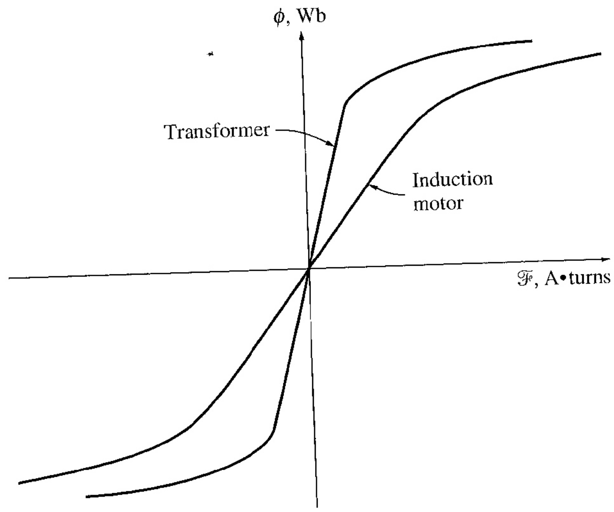
A transformer per-phase equivalent circuit, representing the operation of an induction motor is shown below:



The transformer model or an induction motor, with rotor and stator connected by an ideal transformer of turns ratio a_{eff}.

As in any transformer, there is certain resistance and self-inductance in the primary (stator) windings, which must be represented in the equivalent circuit of the machine. They are - R_1 - stator resistance and X_1 – stator leakage reactance

Also, like any transformer with an iron core, the flux in the machine is related to the integral of the applied voltage E_1. The curve of mmf vs flux (magnetization curve) for this machine is compared to a similar curve for a transformer, as shown below:



The slope of the induction motor's mmf-flux curve is much shallower than the curve of a good transformer. This is because there must be an air gap in an induction motor, which greatly increases the reluctance of the flux path and thus reduces the coupling between primary and secondary windings. The higher reluctance caused by the air gap means that a higher magnetizing current is required to obtain a given flux level. Therefore, the magnetizing reactance X_m in the equivalent circuit will have a much smaller value than it would in a transformer.

The primary internal stator voltage is E_1 is coupled to the secondary E_R by an ideal transformer with an effective turns ratio a_{eff}. The turns ratio for a wound rotor is basically the ratio of the conductors per phase on the stator to the conductors per phase on the rotor. It is rather difficult to see a_{eff} clearly in the cage rotor because there are no distinct windings on the cage rotor.

E_R in the rotor produces current flow in the shorted rotor (or secondary) circuit of the machine.

The primary impedances and the magnetization current of the induction motor are very similar to the corresponding components in a transformer equivalent circuit.

The Rotor Circuit Model

When the voltage is applied to the stator windings, a voltage is induced in the rotor windings. In general, **the greater the relative motion between the rotor and the stator magnetic fields, the greater the resulting rotor voltage and rotor frequency.** The largest relative motion occurs when the rotor is stationary, called the *locked-rotor* or *blocked-rotor* condition, so the largest voltage and rotor frequency are induced in the rotor at that condition. The smallest voltage and frequency occur when the rotor moves at the same speed as the stator magnetic field, resulting in no relative motion.

The magnitude and frequency of the voltage induced in the rotor at any speed between these extremes is directly proportional to the slip of the rotor. Therefore, if the magnitude of the induced rotor voltage at locked-rotor conditions is called E_{R0}, the magnitude of the induced voltage at any slip will be given by:

$$E_R = sE_{R0}$$

And the frequency of the induced voltage at any slip is:

$$f_r = sf_e$$

This voltage is induced in a rotor containing both resistance and reactance. The rotor resistance R_R is a constant, independent of slip, while the rotor reactance is affected in a more complicated way by slip.

The reactance of an induction motor rotor depends on the inductance of the rotor and the frequency of the voltage and current in the rotor. With a rotor inductance of L_R, the rotor reactance is:

$$X_R = \omega_r L_R = 2\pi f_r L_R$$
$$Since\ f_r = sf_e,$$
$$X_R = s2\pi f_e L_R = sX_{R0}$$

where X_{R0} is the blocked rotor reactance. The resulting rotor equivalent circuit is as shown:

The rotor circuit model of an induction motor.

The rotor current flow is:

$$I_R = \frac{E_R}{R_R + jX_R} = \frac{E_R}{R_R + jsX_{R0}} = \frac{E_{R0}}{R_R/s + jX_{R0}}$$

Therefore, the overall rotor impedance talking into account rotor slip would be:

$$Z_{R,eq} = R_R/s + jX_{R0}$$

And the rotor equivalent circuit using this convention is:

The rotor circuit model with all the frequency (slip) effects concentrated in resistor R_R.

In this equivalent circuit, the rotor voltage is a constant E_{R0} V and the rotor impedance $Z_{R,eq}$ contains all the effects of varying rotor slip. Based upon the equation above, at low slips, it can be seen that the rotor resistance is much much bigger in magnitude as compared to X_{R0}. At high slips, X_{R0} will be larger as compared to the rotor resistance.

The Final Equivalent Circuit

To produce the final per-phase equivalent circuit for an induction motor, it is necessary to refer the rotor part of the model over to the stator side. In an ordinary transformer, the voltages, currents and impedances on the secondary side can be referred to the primary by means of the turns ratio of the transformer.

Exactly the same sort of transformation can be done for the induction motor's rotor circuit. If the effective turns ratio of an induction motor is a_{eff} , then the transformed rotor voltage becomes

$$E_1 = E_R^{'} = a_{eff} E_{R0}$$

The rotor current:

$$I_2 = \frac{I_R}{a_{eff}}$$

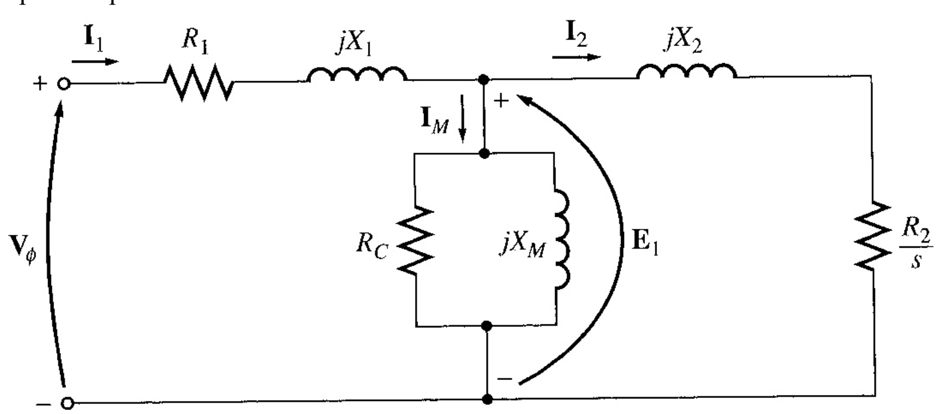
And the rotor impedance:

$$Z_2 = a_{eff}^2 \left(\frac{R_R}{s} + jX_{R0} \right)$$

If we make the following definitions:

$$R_2 = a^2_{eff} R_R$$
$$X_2 = a^2_{eff} X_{R0}$$

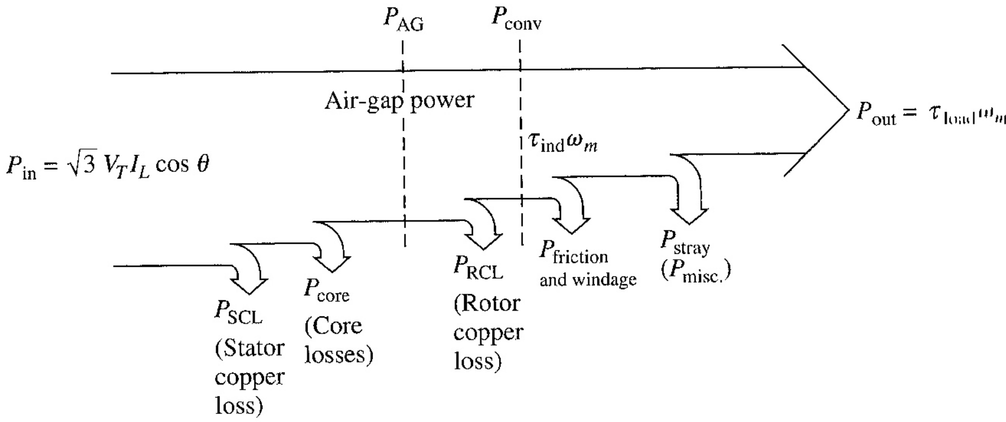
The final per-phase equivalent circuit is as shown below:



4. Power and Torque in Induction Motor

Losses and Power-Flow diagram

An induction motor can be basically described as a rotating transformer. Its input is a 3 phase system of voltages and currents. For an ordinary transformer, the output is electric power from the secondary windings. The secondary windings in an induction motor (the rotor) are shorted out, so no electrical output exists from normal induction motors. Instead, the output is mechanical. The relationship between the input electric power and the output mechanical power of this motor is shown below:



The input power to an induction motor P_{in} is in the form of 3-phase electric voltages and currents. The first losses encountered in the machine are I^2R losses in the stator windings (the stator copper loss P_{SCL}). Then, some amount of power is lost as hysteresis and eddy currents in the stator (P_{core}). The power remaining at this point is transferred to the rotor of the machine across the air gap between the stator and rotor. This power is called the air gap power P_{AG} of the machine. After the power is transferred to the rotor, some of it is lost as I^2R losses (the rotor copper loss P_{RCL}), and the rest is converted from electrical to mechanical form (P_{conv}). Finally, friction and windage losses $P_{F\&W}$ and stray losses P_{misc} are subtracted. The remaining power is the output of the motor P_{out}.

The core losses do not always appear in the power-flow diagram at the point shown in the figure above. Because of the nature of the core losses, where they are accounted for in the machine is somewhat arbitrary. The core losses of an induction motor come partially from the stator circuit and partially from the rotor circuit. Since an induction motor normally operates at a speed near synchronous speed, the relative motion of the magnetic fields over the rotor surface is quite slow, and the rotor core losses are very tiny compared to the stator core losses. Since the largest fraction of the core losses comes from the stator circuit, all the core losses are lumped together at that point on the diagram. These losses are represented in the induction motor equivalent circuit by the resistor R_C (or the conductance G_C). If core losses are just given by a number (X watts) instead of as a circuit element, they are often lumped together with the mechanical losses and subtracted at the point on the diagram where the mechanical losses are located.

The *higher* the speed of an induction motor, the *higher* the friction, windage, and stray losses. On the other hand, the *higher* the speed of the motor (up to n_{sync}), the *lower* its core losses. Therefore, these three categories of losses are sometimes lumped together and called *rotational losses*. The total rotational losses of a motor are often considered to be constant with changing speed, since the component losses change in opposite directions with a change in speed.

Example 7.2

A 480V, 60Hz, 50hp, 3 phase induction motor is drawing 60A at 0.85 PF lagging. The stator copper losses are 2kW, and the rotor copper losses are 700W. The friction and windage losses are 600W, the core losses are 1800W, and the stray losses are negligible. Find:

a) The air gap power P_{AG}
b) The power converted P_{conv}
c) The output power P_{out}
d) The efficiency of the motor

Power and Torque in an Induction Motor

By examining the per-phase equivalent circuit, the power and torque equations governing the operation of the motor can be derived.

The input current to a phase of the motor is: $I_1 = \dfrac{V_\phi}{Z_{eq}}$

Where

$$Z_{eq} = R_1 + jX_1 + \cfrac{1}{G_C - jB_M + \cfrac{1}{\cfrac{V_2}{s} + jX_2}}$$

Thus, the stator copper losses, the core losses, and the rotor copper losses can be found.

The stator copper losses in the 3 phases are: $P_{SCL} = 3\ I_1^2\ R_1$

The core losses : $P_{Core} = 3\ E_1^2\ G_C$

So, the air gap power: $P_{AG} = P_{in} - P_{SCL} - P_{core}$

Also, the only element in the equivalent circuit where the air-gap power can be consumed is in the resistor R_2/s. Thus, the air-gap power:

$$P_{AG} = 3 1_2^2 \frac{R_2}{s}$$

The actual resistive losses in the rotor circuit are given by:

$$P_{RCL} = 3\ I_R^2\ R_R$$

Since power is unchanged when referred across an ideal transformer, the rotor copper losses can also be expressed as:

$$P_{RCL} = 3\ I_2^2\ R_2$$

After stator copper losses, core losses and rotor copper losses are subtracted from the input power to the motor, the remaining power is converted from electrical to mechanical form. The power converted, which is called developed mechanical power is given as:

$$\begin{aligned} P_{conv} &= P_{AG} - P_{RCL} \\ &= 3I_2^2 \frac{R_2}{s} - 3I_2^2 R_2 \\ &= 3I_2^2 R_2 \left(\frac{1}{s} - 1\right) \\ P_{conv} &= 3I_2^2 R_2 \left(\frac{1-s}{s}\right) \end{aligned}$$

And the rotor copper losses are noticed to be equal to the air gap power times the slip → $P_{RCL} = s\ P_{AG}$

Hence, the lower the slip of the motor, the lower the rotor losses. Also, if the rotor is not turning, the slip is s=1 and the air gap power is entirely consumed in the rotor. This is logical, since if the rotor is not turning, the output power P_{out} ($= \tau_{load}\,\omega_m$) must be zero. Since $P_{conv} = P_{AG} - P_{RCL}$, this also gives another relationship between the air-gap power and the power converted from electrical and mechanical form:

$$P_{conv} = P_{AG} - P_{RCL}$$

$$= P_{AG} - sP_{AG}$$

$$P_{conv} = (1\text{-}s)\,P_{AG}$$

Finally, if the friction and windage losses and the stray losses are known, the output power:

$$P_{out} = P_{conv} - P_{F\&W} - P_{misc}$$

The induced torque in a machine was defined as the torque generated by the internal electric to mechanical power conversion. This torque differs from the torque actually available at the terminals of the motor by an amount equal to the friction and windage torques in the machine. Hence, the developed torque is:

$$\tau_{ind} = \frac{P_{conv}}{\omega_m}$$

Other ways to express torque:

$$\tau_{ind} = \frac{(1-s)P_{AG}}{(1-s)\omega_{sync}}$$

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

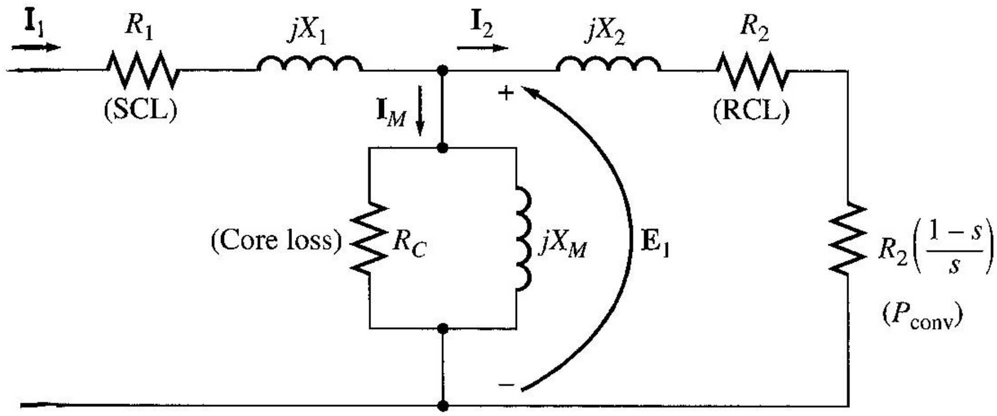
Separating the Rotor Copper Losses and the Power Converted in an Induction Motor's Equivalent Circuit

A portion of power transferred via the air gap will be consumed by the rotor copper loss and also converted into mechanical power. Hence it may be useful to separate the rotor copper loss element since rotor resistance are both used for calculating rotor copper loss and also the output power.

Since Air Gap power would require R_2/s and rotor copper loss require R_2 element. The difference between the air gap power and the rotor copper loss would give the converted power, hence;

$$R_{conv} = \frac{R_2}{s} - R_2 = R_2\left(\frac{1-s}{s}\right)$$

Therefore the equivalent circuit would be modified to be as follows:



Example 7.3

A 460V, 25hp, 60Hz, 4 pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$R_1 = 0.641\ \Omega$	$R_2 = 0.332\ \Omega$	
$X_1 = 1.106\ \Omega$	$X_2 = 0.464\ \Omega$	$X_m = 26.3\ \Omega$

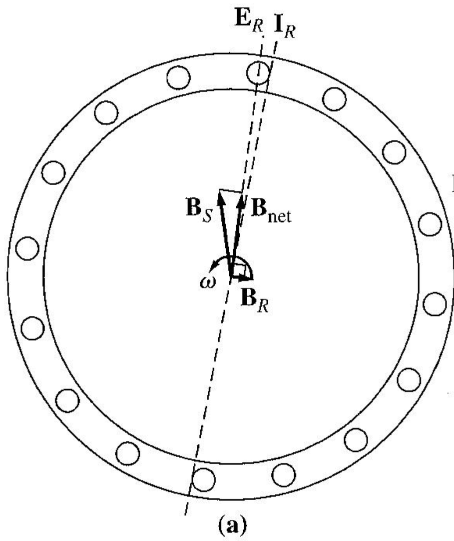
The total rotational losses are 1100W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2% at the rated voltage and rated frequency, find the motor's

a) speed
b) stator current
c) power factor
d) P_{conv} and P_{out}
e) τ_{ind} and τ_{load}
f) efficiency

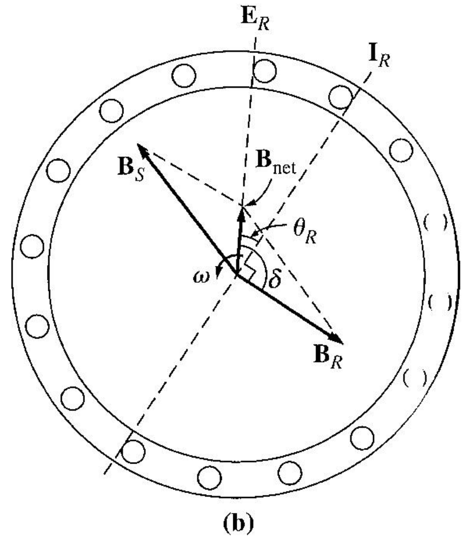
5. Induction Motor Torque-Speed Characteristics

The torque-speed relationship will be examined first from the physical viewpoint of the motor's magnetic filed behaviour and then, a general equation for torque as a function of slip will be derived from the induction motor equivalent circuit.

Induced Torque from a Physical Standpoint



The magnetic fields in an induction motor under **light loads**



The magnetic fields in an induction motor under **heavy loads**

No-load Condition

Assume that the induction rotor is already rotating at no load conditions, hence its rotating speed is near to synchronous speed. The net magnetic field B_{net} is produced by the magnetization current I_M . The magnitude of I_M and B_{net} is directly proportional to voltage E_1 . If E_1 is constant, then B_{net} is constant. In an actual machine, E_1 varies as the load changes due to the stator impedances R_1 and X_1 which cause varying volt drops with varying loads. However, the volt drop at R_1 and X_1 is so small, that E_1 is assumed to remain constant throughout.

At no-load, the rotor slip is very small, and so the relative motion between rotor and magnetic field is very small, and the rotor frequency is also very small. Since the relative motion is small, the voltage E_R induced in the bars of the rotor is very small, and the resulting current flow I_R is also very small. Since the rotor frequency is small, the reactance of the rotor is nearly zero, and the max rotor current I_R is almost in phase with the rotor voltage E_R . The rotor current produces a small magnetic field B_R at an angle slightly greater than 90 degrees behind B_{net}. The stator current must be quite large even at no-load since it must supply most of B_{net} .

The induced torque which is keeping the rotor running, is given by:

$$\tau_{ind} = kB_R \times B_{net}$$

and its magnitude is $\tau_{ind} = kB_R B_{net} \sin\delta$

In terms of magnitude, the induced torque will be small due to small rotor magnetic field.

On-load Conditions

As the motor's load increases, its slip increases, and the rotor speed falls. Since the rotor speed is slower, there is now more relative motion between rotor and stator magnetic fields. Greater relative motion means a stronger rotor voltage E_R which in turn produces a larger rotor current I_R . With large rotor current, the rotor magnetic field B_R also increases. However, the angle between rotor current and B_R changes as well.

Since the rotor slip is larger, the rotor frequency rises ($f_r = sf_e$) and the rotor reactance increases (ωL_R). Therefore, the rotor current now lags further behind the rotor voltage, and the rotor magnetic field shifts with the current. The rotor current now has increased compared to no-load and the angle δ has increased. The increase in B_R tends to increase the torque, while the increase in angle δ tends to decrease the torque (τ_{ind} is proportional to sin δ, and $\delta > 90°$). Since the first effect is larger than the second one, the overall induced torque increases to supply the motor's increased load.

As the load on the shaft is increased, the sin δ term decreases more than the B_R term increases (the value is going towards the 0 cross over point for a sine wave). At that point, a further increase in load decreases τ_{ind} and the motor stops. This effect is known as **pullout torque**.

Modelling the torque-speed characteristics of an induction motor

Looking at the induction motor characteristics, a summary on the behaviour of torque:

Note: $\tau_{ind} = kB_R B_{net} \sin\delta$

a) Rotor magnetic field will increase as the rotor current will increase (provided that the rotor core is not saturated). Current flow will increase as slip increase (reduction in velocity)
b) The net magnetic field density will remain constant since it is proportional to E_1 (refer to equivalent induction motor equivalent circuit). Since E_1 is assumed to be constant, hence B_{net} will assume to be constant.
c) The angle δ will increase as slip increases. Hence the sin δ value will reduce until as such that the reduction of sin d will be greater than the increase of B_R (pullout torque). Since δ is greater than 90 degrees, as such that:

$$\sin\delta = \sin\left(\theta_r + 90°\right) = \cos\theta_r$$

where:
θ_r is the angle between E_R and I_R (note that E_R is in phase with B_{net} since it is in phase with B_{net}).

Adding the characteristics of all there elements would give the torque speed characteristics of an induction motor.

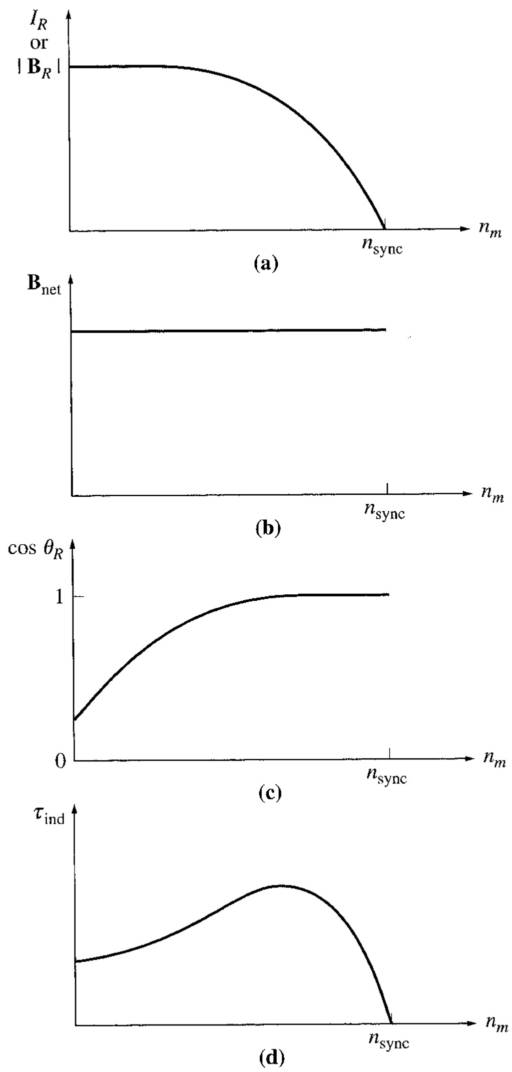
cos θ_R can also be known as the motor power factor where:

$$\theta_r = \tan^{-1}\frac{X_r}{R_r} = \tan^{-1}\frac{sX_o}{R_r}$$

The torque speed curve may be divided into 3 regions of operations:

a) Linear region or low slip region
b) Moderate slip region located until the pullout torque level.
c) High slip region

Typical values of pullout torque would be at about 200% to 250% of the rated full load torque of the induction machine. The starting torque would be about 150% than the rated full load torque; hence induction motor may be started at full load.



Graphical development of an induction motor torque-speed characteristics

The Derivation of the Induction Motor Induced-Torque Equation

Previously we looked into the creation of the induced torque graph, now we would like to derive the Torque speed equation based upon the power flow diagram of an induction motor. We know that,

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} \qquad or \qquad \tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

Comparing between the 2 equations, the second equation may be more useful since it is referenced to synchronous speed. Hence there is a need to derive P_{AG}. By definition, air gap power is the power transferred from the stator to the rotor via the air gap in the induction machine. Based upon the induction motor equivalent circuit, the air gap power may be defined as:

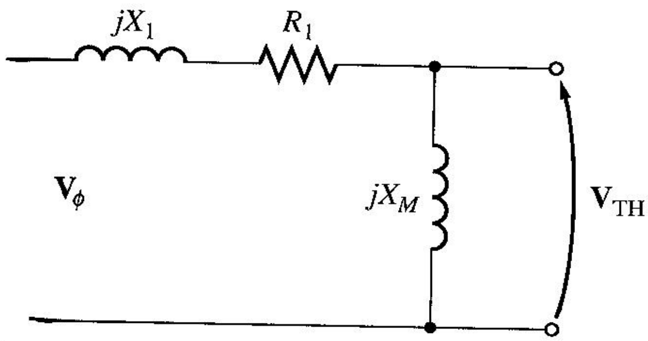
$$P_{AG\ per\ phase} = I_2^2 \frac{R_2}{s}$$

hence, total air gap power:

$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$

Our next task is to find I_2 (current flow in the rotor circuit). The easiest way is via the construction of the Thevenin equivalent circuit.
Thevenin's theorem states that any linear circuit that can be separated by two terminals from the rest of the system can be replaced by a single voltage source in series with an equivalent impedance.

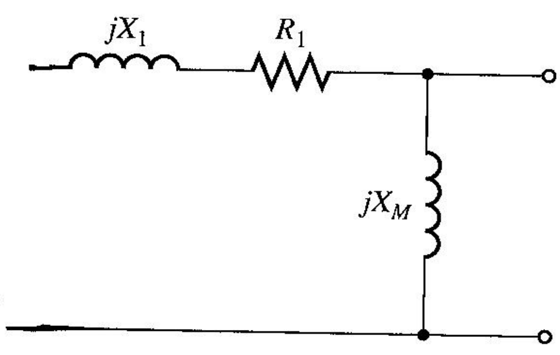
Calculation via thevenin equivalent method



$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + jX_1 + jX_M}\mathbf{V}_\phi$$

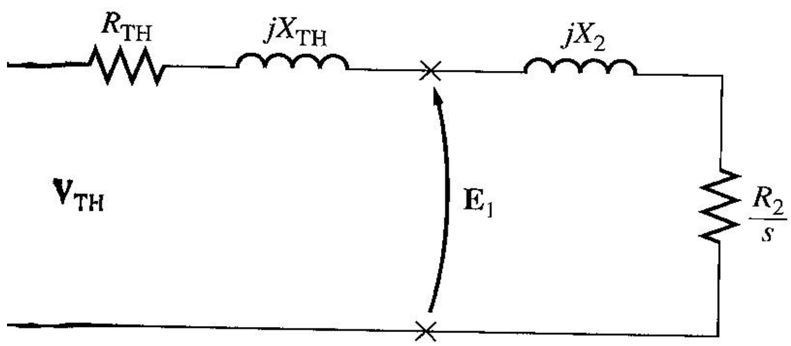
$$V_{TH} = \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}} V_\phi$$

(a)



$$Z_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

(b)



(c)

1) Derive the thevenin voltage (potential divider rule):

$$V_{TH} = V_{\phi} \frac{jX_m}{R_1 + jX_1 + jX_m}$$

Hence the magnitude of thevenin voltage:

$$V_{TH} = V_{\phi} \frac{X_m}{\sqrt{R_1^2 + \left(X_1 + X_m\right)^2}}$$

Since Xm >> X1 , Xm >> R1, therefore the magnitude may be approximated to:

$$V_{TH} \approx V_{\phi} \frac{X_m}{X_1 + X_m}$$

2) Find the thevenin impedance

Take out the source and replace it with a short circuit, and derive the equivalent impedances.

$$Z_{TH} = \frac{jX_m\left(R_1 + jX_1\right)}{R_1 + jX_1 + jX_m}$$

Since Xm >> X1, Xm >> R1,

$$R_{TH} \approx R_1 \left(\frac{X_m}{X_1 + X_m}\right)^2$$

$$X_{TH} \approx X_1$$

Representing the stator circuit by the thevenin equivalent, and adding back the rotor circuit, we can derive I2,

$$I_2 = \frac{V_{TH}}{R_{TH} + R_2/s + j(X_{TH} + X_2)}$$

Hence the magnitude will be,

$$I_2 = \frac{V_{TH}}{\sqrt{\left(R_{TH} + R_2/s\right)^2 + \left(X_{TH} + X_2\right)^2}}$$

Hence air gap power,

$$P_{AG} = 3\left(\frac{V_{TH}}{\sqrt{\left(R_{TH} + {R_2}/{s}\right)^2 + \left(X_{TH} + X_2\right)^2}}\right)^2 \frac{R_2}{s}$$

Hence, induced torque,

$$\tau_{ind} = \frac{3\left(\frac{V_{TH}}{\sqrt{\left(R_{TH} + {R_2}/{s}\right)^2 + \left(X_{TH} + X_2\right)^2}}\right)^2 \frac{R_2}{s}}{\omega_{sync}}$$

If a graph of Torque and speed were plotted based upon changes in slip, we would get a similar graph as we had derived earlier.

<u>Comments on the Induction Motor Torque Speed Curve</u>

a) Induced Torque is zero at synchronous speed.
b) The graph is nearly linear between no load and full load (at near synchronous speeds).
c) Max torque is known as pull out torque or breakdown torque
d) Starting torque is very large.
e) Torque for a given slip value would change to the square of the applied voltage.
f) If the rotor were driven faster than synchronous speed, the motor would then become a generator.
g) If we reverse the direction of the stator magnetic field, it would act as a braking action to the rotor – **plugging**.

Since P_{conv} may be derived as follows:

$$P_{conv} = \tau_{ind}\omega_m$$

Hence we may plot a similar characteristic to show the amount of power converted throughout the variation of load.

<u>Maximum (Pullout) Torque in an Induction Motor</u>

Since induced torque is equal to P_{AG} / ω_{sync} , the maximum pullout torque may be found by finding the maximum air gap power. And maximum air gap power is during which the power consumed by the R_2/s resistor is the highest.

Based upon the maximum power transfer theorem, maximum power transfer will be achieved when the magnitude of source impedance matches the load impedance. Since the source impedance is as follows:

$$Z_{source} = R_{TH} + jX_{TH} + jX_2$$

Hence maximum power transfer occurs during:

$$\frac{R_2}{s} = \sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}$$

Hence max power transfer is possible when slip is as follows:

$$s_{\max} = \frac{R_2}{\sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}}$$

Put in the value of S_{max} into the torque equation,

$$\tau_{\max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}\right]}$$

From here we can say:

a) Torque is related to the square of the applied voltage
b) Torque is also inversely proportional to the machine impedances
c) Slip during maximum torque is dependent upon rotor resistance
d) Torque is also independent to rotor resistance as shown in the maximum torque equation.

By adding more resistance to the machine impedances, we can vary:

a) Starting torque
b) Max pull out speed

<u>Example 7.4</u>

A 2 pole, 50 Hz induction motor supplies 15kW to a load at a speed of 2950 r/min.

(a) What is the motor's slip?
(b) What is the induced torque in the motor in Nm under these conditions?
(c) What will the operating speed of the motor be if its torque is doubled?
(d) How much power will be supplied by the motor when the torque is doubled?

<u>Example 7.5</u>

A 460V, 25hp, 60Hz, 4-pole, Y-connected wound rotor induction motor has the following impedances in ohms per-phase referred to the stator circuit:

$R_1 = 0.641\ \Omega$ $R_2 = 0.332\ \Omega$
$X_1 = 1.106\ \Omega$ $X_2 = 0.464\ \Omega$ $X_m = 26.3\ \Omega$

(a) What is the max torque of this motor? At what speed and slip does it occur?
(b) What is the starting torque?
(c) When the rotor resistance is doubled, what is the speed at which the max torque now occurs? What is the new starting torque?

6. Variations in Induction Motor Torque-Speed Characterictics

HIGH resistance rotor
↓
Starting torque HIGH
↓
Slip HIGH at normal conditions
↓
$P_{conv} = (1\text{-}s)\, P_{AG}$
S increase, P_{AG} decrease, and **efficiency decrease**.

LOW resistance rotor
↓
Starting torque LOW, HIGH starting current
↓
Slip LOW at normal conditions
↓
$P_{conv} = (1\text{-}s)\, P_{AG}$
S decrease, P_{AG} increase, and **efficiency increase**.

↓

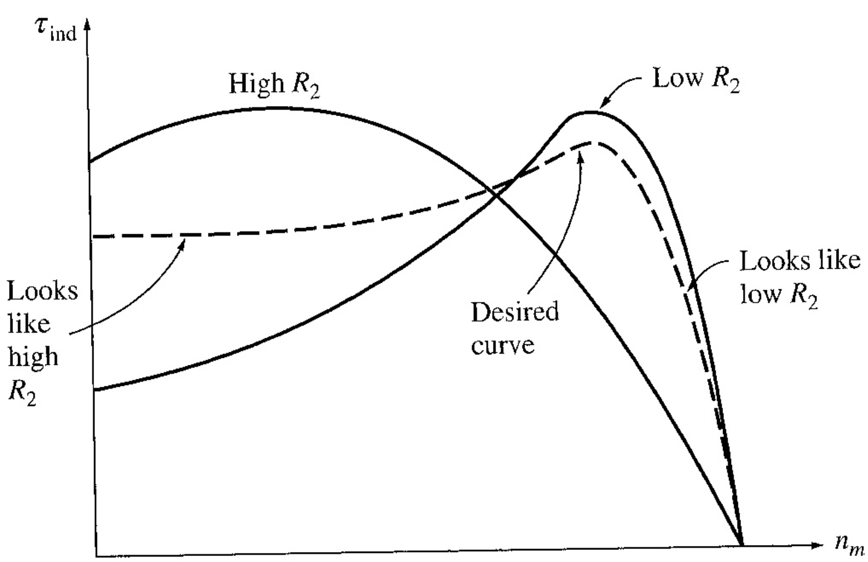
Use a wound rotor induction motor and insert extra resistance into the rotor during starting, and then removed for better efficiency during normal operations.

↓

But, wound rotor induction motors are more expensive, need more maintanence etc.

↓

Solution - utilising leakage reactance – to obtain the desired curve as shown below

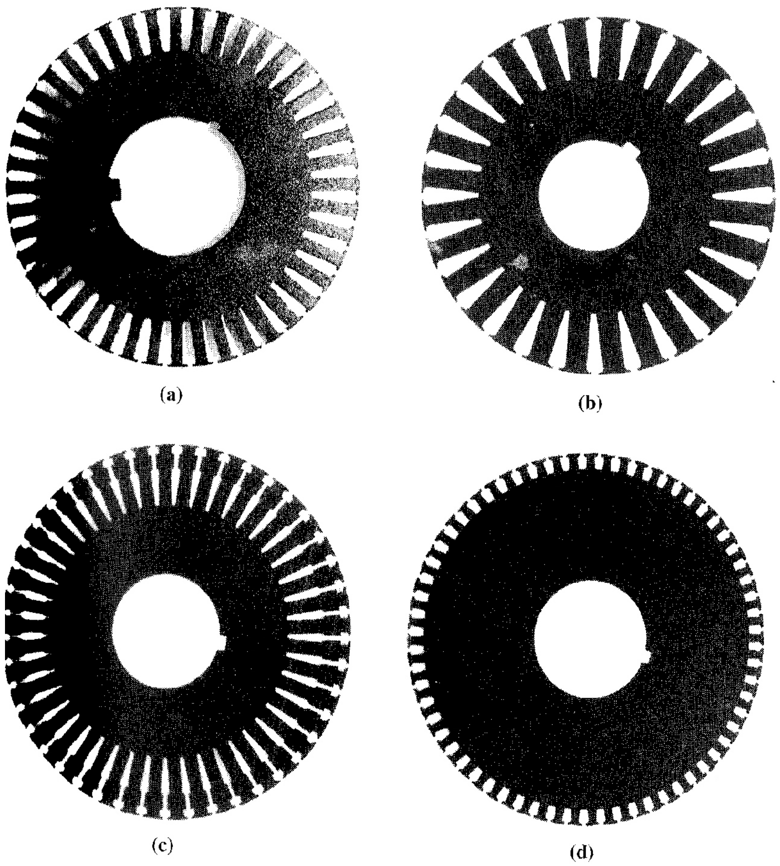


A torque-speed characteristic curve combining high-resistance effects at low speeds (high slip) with low resistance effects at high speed (low slip).

Control of Motor Characteristics by Cage Rotor Design

Leakage reactance X_2 represents the referred form of the rotor's leakage reactance (reactance due to the rotor's flux lines that do not couple with the stator windings.)

Generally, the farther away the rotor bar is from the stator, the greater its X_2, since a smaller percentage of the bar's flux will reach the stator. Thus, if the bars of a cage rotor are placed near the surface of the rotor, they will have small leakage flux and X_2 will be small.



Laminations from typical cage induction motor, cross section of the rotor bars

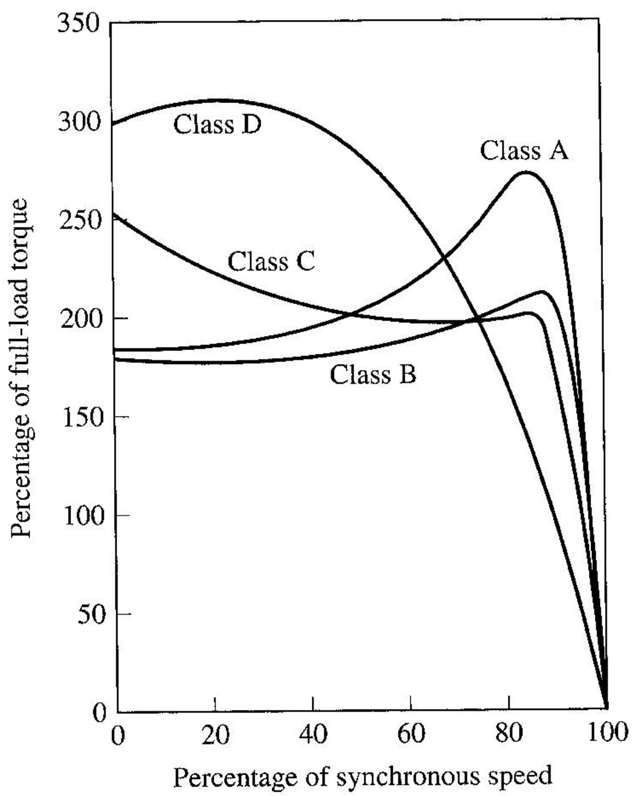
(a) NEMA class A – large bars near the surface
(b) NEMA class B – large, deep rotor bars
(c) NEMA class C – double-cage rotor design
(d) NEMA class D – small bars near the surface

NEMA (National Electrical Manufacturers Association) class A

- Rotor bars are quite large and are placed near the surface of the rotor.
- Low resistance (due to its large cross section) and a low leakage reactance X_2 (due to the bar's location near the stator)
- Because of the low resistance, the pullout torque will be quite near synchronous speed
- Motor will be quite **efficient**, since little air gap power is lost in the rotor resistance.
- However, since R_2 is small, **starting torque will be small**, and starting current will be high.
- This design is the standard motor design.
- Typical applications – driving fans, pumps, and other machine tools.

NEMA class D

- Rotor with small bars placed near the surface of the rotor (higher-resistance material)
- High resistance (due to its small cross section) and a low leakage reactance X_2 (due to the bar's location near the stator)
- Like a wound-rotor induction motor with extra resistance inserted into the rotor.
- Because of the large resistance, the pullout torque occurs at high slip, and **starting torque** will be quite **high**, and low starting current.
- Typical applications – extremely high-inertia type loads.

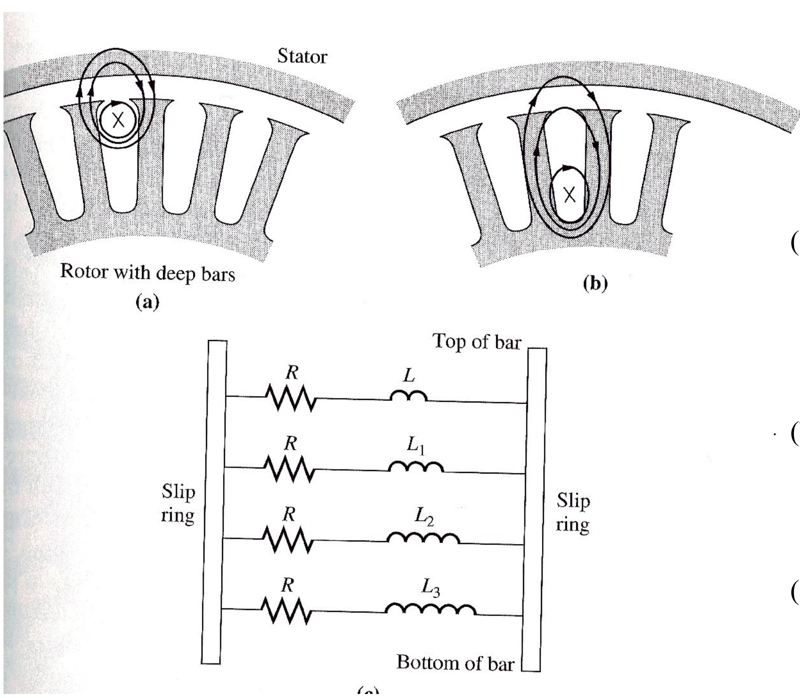


Typical torque-speed curves for different rotor designs.

Deep-Bar and Double-Cage rotor design

How can a variable rotor resistance be produced to combine the high starting torque and low starting current of Class D, with the low normal operating slip and high efficiency of class A??
→ Use deep rotor bars (Class B) or double-cage rotors (Class C)

The basic concept is illustrated below:



(a) For a current flowing in the top of the bar, the flux is tightly linked to the stator, and leakage L is small.
(b) At the bottom of the bar, the flux is loosely linked to the stator, and leakage L is large.
(c) Resulting equivalent circuit

NEMA Class B

- At the upper part of a deep rotor bar, the current flowing is tightly coupled to the stator, and hence the leakage inductance is small in this region. Deeper in the bar, the leakage inductance is higher.

- At **low slips**, the rotor's frequency is very small, and the reactances of all the parallel paths are small compared to their resistances. The impedances of all parts of the bar are approx equal, so current flows through all the parts of the bar equally. The resulting large cross sectional area makes the rotor resistance quite small, resulting in good efficiency at low slips.

- At **high slips** (starting conditions), the reactances are large compared to the resistances in the rotor bars, so all the current is forced to flow in the low-reactance part of the bar near the stator. Since the effective cross section is lower, the rotor resistance is higher. Thus, the starting torque is relatively higher and the starting current is relatively lower than in a class A design.

- Applications similar to class A, and this type B have largely replaced type A.

NEMA Class C

- It consists of a large, low resistance set of bars buried deeply in the rotor and a small, high-resistance set of bars set at the rotor surface. It is similar to the deep-bar rotor, except that the difference between low-slip and high-slip operation is even more exaggerated.

- At starting conditions, only the small bars are effective, and the rotor resistance is high. Hence, high starting torque. However, at normal operating speeds, both bars are effective, and the resistance is almost as low as in a deep-bar rotor.

- Used in high starting torque loads such as loaded pumps, compressors, and conveyors.

NEMA Class E and F

- Class E and Class F are already discontinued. They are low starting torque machines.

7. Starting Induction Motors

An induction motor has the ability to start directly, however direct starting of an induction motor is not advised due to high starting currents, which will be explained later.

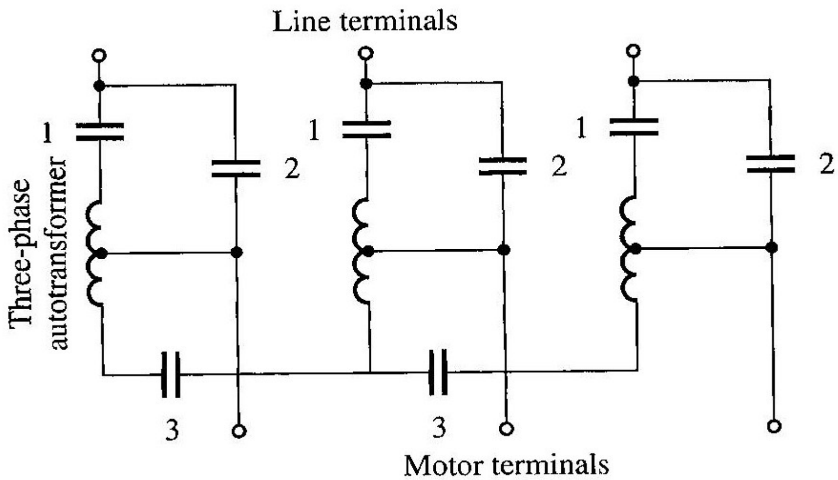
In order to know the starting current, we should be able to calculate the starting power required by the induction motor. The Code Letter designated to each induction motor, which can be seen in figure 7-34, may represent this. (The starting code may be obtained from the motor nameplate).

$$S_{start} = rated\ horsepower \times code\ letter$$

$$I_L = \frac{S_{start}}{\sqrt{3}V_T}$$

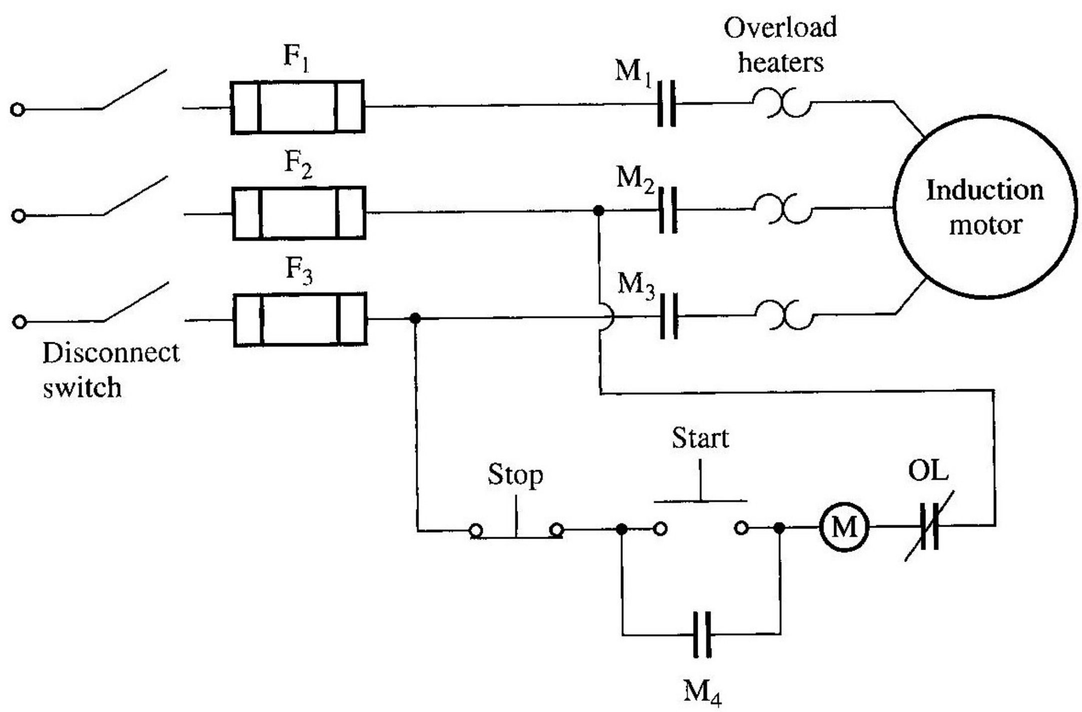
Based upon example7-7, it is seen that to start an induction motor, there is a need for high starting current. For a wound rotor type induction motor, this problem may be solved by incorporating resistor banks at the rotor terminal during starting (to reduce current flow) and as the rotor picks up speed, the resistor banks are taken out.

For a squirrel cage rotor, reducing starting current may be achieved by varying the starting voltage across the stator terminal. Reducing the starting terminal voltage will also reduce the rated starting power hence reducing starting current. One way to achieve this is by using a step down transformer during the starting sequence and stepping up the transformer ratio as the machine spins faster (refer figure below).



Starting sequence:
(a) Close 1 and 3
(b) Open 1 and 3
(c) Close 2

Induction motor starting circuits



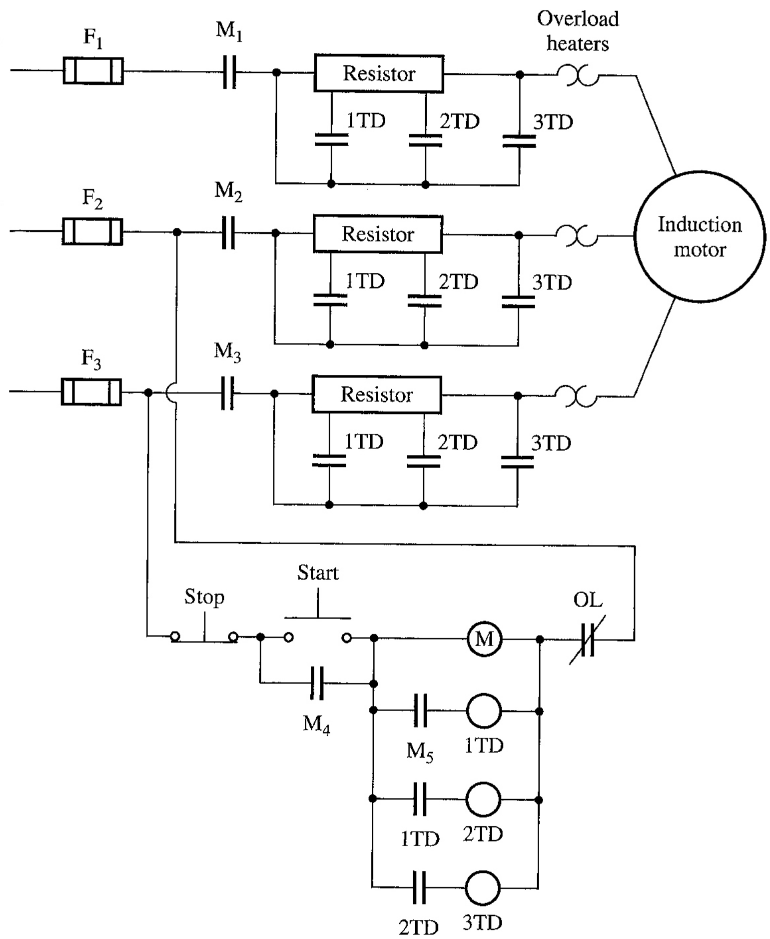
Operation:

- When the start button is pressed, the relay or contactor coil M is energized, causing the normally open contacts M1, M2 and M3 to shut.
- Then, power is applied to the induction motor, and the motor starts.
- Contact M4 also shuts which shorts out the starting switch, allowing the operator to release it without removing power from the M relay.
- When the stop button is pressed, the M relay is deenergized, and the M contacts open, stopping the motor.

A magnetic motor starter of this sort has several built in protective features:

a) Short Circuit protection – provided by the fuses
b) Overload protection – provided by the Overload heaters and the overload contacts (OL)
c) Undervoltage protection – deenergising of the M relays.

<u>3 Step resistive Starter Induction motor</u>



Operation:

- Similar to the previous one, except that there are additional components present to control removal of the starting resistor. Relays 1TD, 2TD and 3TD are called time-delay relays.
- When the start button is pushed, the M relay energizes and power is applied to the motor.
- Since the 1TD, 2TD and 3TD contacts are all open, the full starting resistor are in series with the motor, reducing the starting current.
- When M contacts close, the 1TD relay is energized. There is a finite delay before the 1TD contacts close. During that time, the motor speeds up, and the starting current drops.
- After that, 1TD close, cutting out part of the starting resistance and simultaneously energizing 2TD relay. And finally 3TD contacts close, and the entire starting resistor is out of the circuit.

8. Speed Control of Induction Motor

Induction motors are not good machines for applications requiring considerable speed control. The normal operating range of a typical induction motor is confined to less than 5% slip, and the speed variation is more or less proportional to the load.

Since $P_{RCL} = sP_{AG}$, if slip is made higher, rotor copper losses will be high as well.

There are basically 2 general methods to control induction motor's speed:

a) Varying stator and rotor magnetic field speed
b) Varying slip

Varying the magnetic field speed may be achieved by varying the **electrical frequency** or by **changing the number of poles**.

Varying slip may be achieved by **varying rotor resistance** or **varying the terminal voltage**.

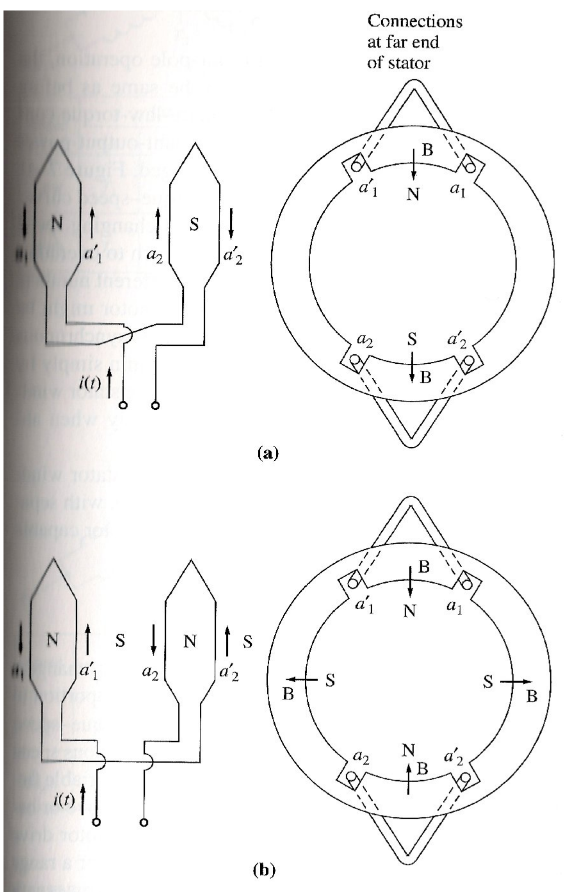
Induction Motor Speed Control by Pole Changing

There are 2 approaches possible:

a) Method of Consequent Poles (Old Method)
b) Multiple Stator Windings Method

Method of Consequent Poles

General Idea:
Consider one phase winding in a stator. By changing the current flow in one portion of the stator windings as such that it is similar to the current flow in the opposite portion of the stator will automatically generate an extra pair of poles.



(a) In the 2-pole configuration, one coil is a north pole and the other a south pole.
(b) When the connection on one of the 2 coils is reversed, they are both north poles, and the south poles are called consequent poles, and the windings is now a four - pole windings.

By applying this method, the number of poles may be maintained (no changes), doubled or halfed, hence would vary its operating speed.

In terms of torque, the maximum torque magnitude would generally be maintained.

Disadvantage:

This method will enable speed changes in terms of 2:1 ratio steps, hence to obtained variations in speed, multiple stator windings has to be applied. Multiple stator windings have extra sets of windings that may be switched in or out to obtain the required number of poles. Unfortunately this would an expensive alternative.

Speed Control by Changing the Line Frequency

Changing the electrical frequency will change the synchronous speed of the machine.

Changing the electrical frequency would also require an adjustment to the terminal voltage in order to maintain the same amount of flux level in the machine core. If not the machine will experience:

a) Core saturation (non linearity effects)
b) Excessive magnetization current.

Varying frequency with or without adjustment to the terminal voltage may give 2 different effects:

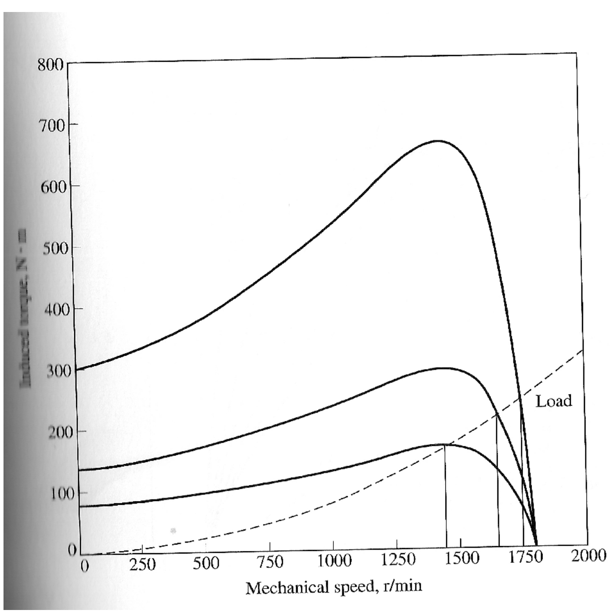
a) Vary frequency, stator voltage adjusted – generally vary speed and maintain operating torque.
b) Vary Frequency, stator voltage maintained – able to achieve higher speeds but a reduction of torque as speed is increased.

There may also be instances where both characteristics are needed in the motor operation; hence it may be combined to give both effects.

With the arrival of solid-state devices/power electronics, line frequency change is easy to achieved and it is more versatile to a variety of machines and application.

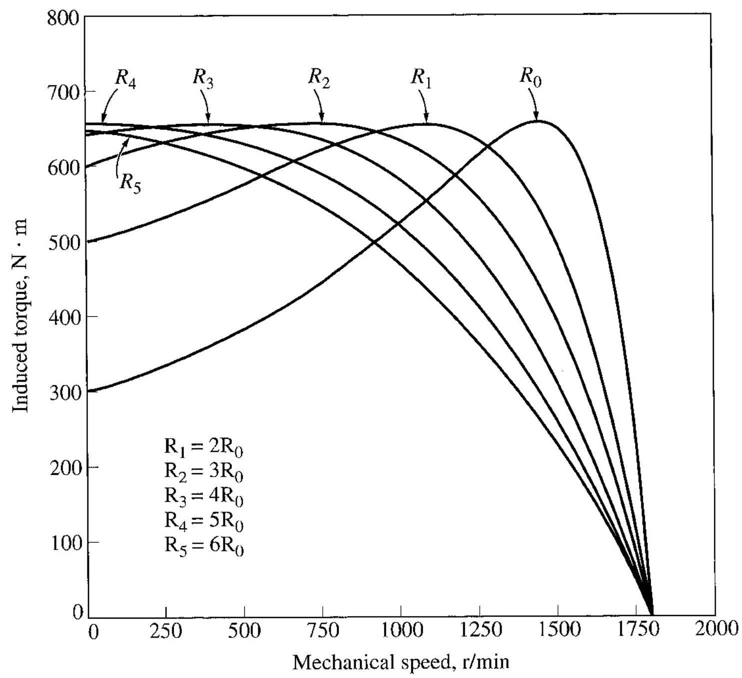
Speed Control by Changing the Line Voltage

Varying the terminal voltage will vary the operating speed but with also a variation of operating torque. In terms of the range of speed variations, it is not significant hence this method is only suitable for small motors only.



Speed Control by Changing the Rotor Resistance

It is only possible for wound rotor applications but with a cost of reduced motor efficiency.



9. Determining Circuit Model Parameters

There are basically 3 types of tests that can be done on an Induction motor:

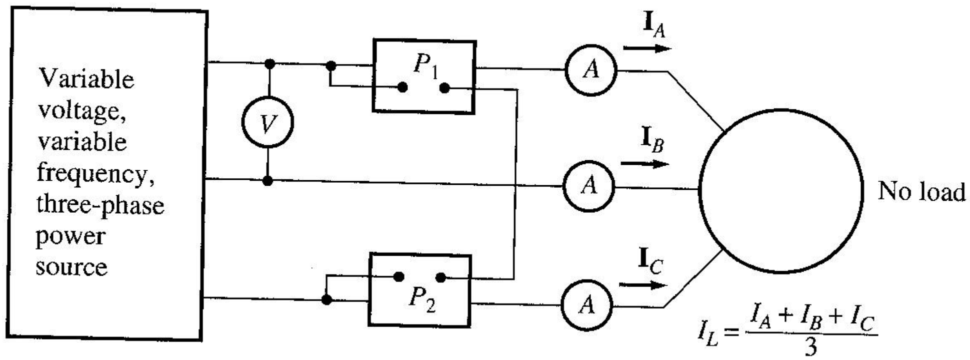
a) No-load test
b) DC test
c) Locked Rotor test or Blocked Rotor test

These tests are performed to determine the equivalent circuit elements – R_1, R_2, X_1, X_2 and X_M.

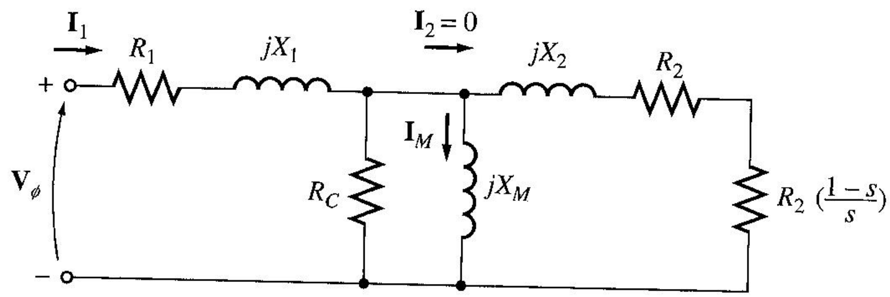
<u>The No-Load Test</u>

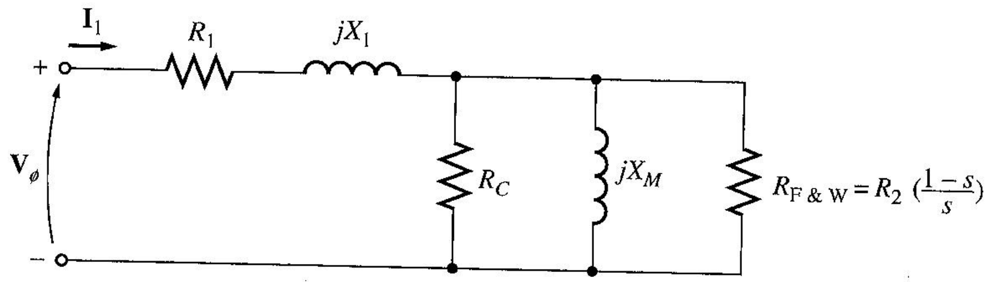
The no-load test measures the rotational losses and provides info about its magnetization current.

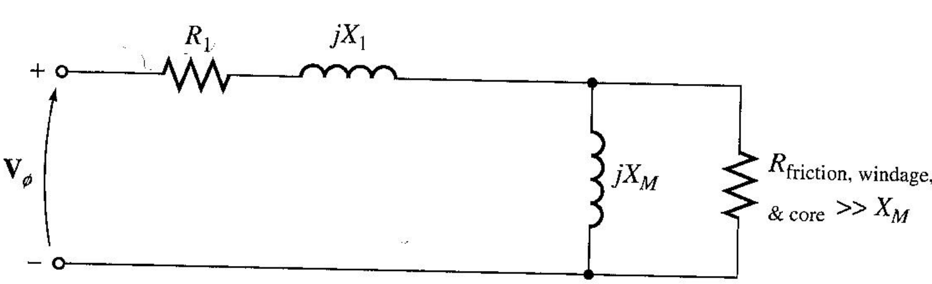
The induction motor is not loaded; hence any load will be based upon frictional and mechanical losses. The rotor will be rotating at near synchronous speed hence slip is very small. The no load test circuit and induction motor equivalent circuit is shown below:



(a)







With its very small slip, the resistance corresponding to its power converted, $R_2(1\text{-}s)/s$, is much larger than the resistance corresponding to the rotor copper losses R_2 and much larger than the rotor reactance X_2.

In this case, the equivalent circuit reduces to the last circuit. There, the output resistor is in parallel with the magnetization reactance X_M and the core losses R_C.

In this motor at no-load conditions, the input power measured by the meters must equal the losses in the motor. The rotor copper losses are negligible because I_2 is extremely small (because of the large load resistance $R_2(1\text{-}s)/s$), so they may be neglected. The stator copper loss is given by:

$$P_{SCL} = 3I_1^2 R_1$$

Hence,

$$\begin{aligned} P_{IN} &= P_{SCL} + P_{CORE} + P_{F\&W} + P_{MISC} \\ &= 3I_1^2 R_1 + P_{ROT} \end{aligned}$$

$$P_{rot} = P_{core} + P_{F\&W} + P_{misc}$$

Also,

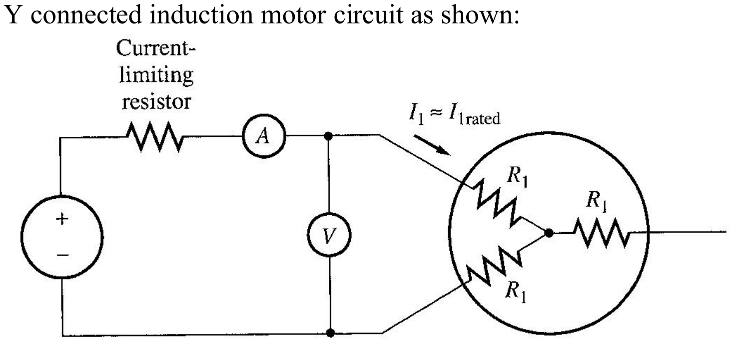
$$\left|Z_{eq}\right| = \frac{V_\phi}{I_{1,nl}} \approx X_1 + X_m$$

<u>The DC Test</u>

This is a test for R_1 independent of R_2, X_1, and X_2.

DC voltage is applied to the terminals of the stator windings of the induction motor. Since it is DC supply, f = 0, hence no induced current in the rotor circuit. Current will flow through the stator circuit. Reactance is zero at dc. Thus, the only quantity limiting current flow in the motor is the stator resistance, and it can be determined.

Assume we have a Y connected induction motor circuit as shown:



Steps:

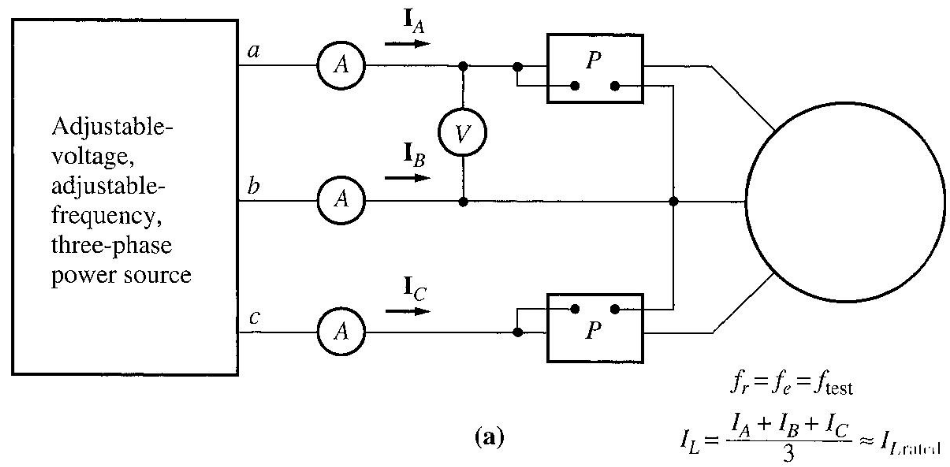
a) DC voltage is applied across the motor terminal and current flow is adjusted to rated condition (to simulate normal operating condition)
b) Voltage and current flow is noted.

Based upon the test configuration,

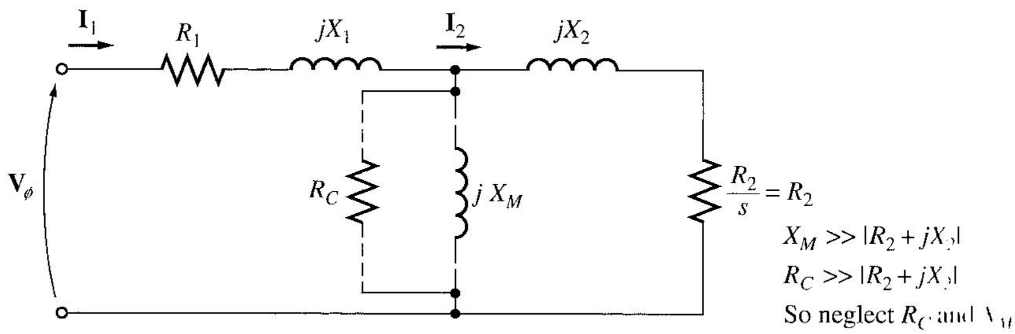
$$2R_1 = \frac{V_{DC}}{I_{DC}} \quad , \quad \therefore R_1 = \frac{V_{DC}}{2I_{DC}}$$

Since we are able to determine the value of R1, hence P_{SCL} can be calculated. Unfortunately, this method is not accurate since it is done using a DC power source where skin effects, that occurs when an ac voltage is applied to the windings, are neglected.

The Locked-Rotor Test



(a)



(b)

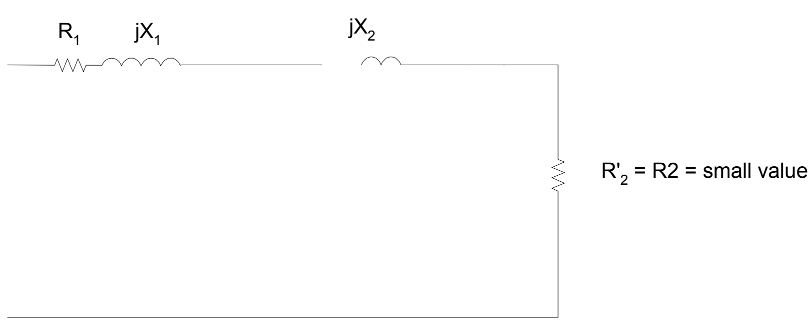
Steps:

a) The rotor is locked.

b) AC voltage is applied across the stator terminals and current flow is adjusted to full load condition.

c) Measure voltage, current and power flow.

Since the rotor is locked, hence slip would be at a maximum as such that the R_2 terms are small. Hence bulk of the current will flow through the rotor circuit rather than the magnetizing branch. Therefore the overall circuit is reduced to:



From here we may calculate:

$$P_{IN} = \sqrt{3}V_T I_L \cos\theta \quad , \qquad PF = \cos\theta = \frac{P_{IN}}{\sqrt{3}V_T I_L}$$

Also,

$$|Z_{LR}| = \frac{V_\phi}{I_1} = \frac{V_T}{\sqrt{3}I_L} = R_{LR} + jX'_{LR} = |Z_{LR}|\cos\theta + j|Z_{LR}|\sin\theta$$

$$Note: R_{LR} = R_1 + R_2 \quad , \qquad X'_{LR} = X'_1 + X'_2$$

Note: This test is generally inaccurate due to the fact that in real operation, slip would vary from starting and as the rotor approaches operating speeds. Since slip would also correlate to rotor current and voltage frequency (at small slip, frequency is small, at high slip, frequency is high). Frequency would affect the rotor reactance. Therefore, this test is done with a lower supply frequency (25% or less) to simulate small slip during operation. However, the true value of X may be found in the following formulae:

Since R1 may be found from the DC test, therefore we can calculate R2. The value of X_{LR} may also be calculated by using the formulae below:

$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = X_1 + X_2$$

Example 7-8

The following test data were taken on a 7.5 hp, 4-pole, 208V, 60Hz, desing A, Y-connected induction motor having a rated current of 28A.

DC Test: V_{DC} = 13.6 V I_{DC} = 28.0 A

No-load test:

V_T = 208 V	f = 60 Hz
I_A = 8.12 A	P_{in} = 420W
I_B = 8.20 A	
I_C = 8.18 A	

Locked-rotor test:

V_T = 25 V	f = 15 Hz
I_A = 28.1 A	P_{in} = 920W
I_B = 28.0 A	
I_C = 27.6 A	

(a) Sketch the per-phase equivalent circuit for this motor
(b) Find the slip at the pullout torque, and find the value of the pullout torque.

CHAPTER 8 – DC MACHINERY FUNDAMENTALS

Summary:

1. **A Simple Rotating Loop between Curved Pole Faces**
   - The Voltage Induced in a Rotating Loop
   - Getting DC voltage out of the Rotating Loop
   - The Induced Torque in the Rotating Loop

2. **Commutation in a Simple Four-Loop DC Machine**

3. **Problems with Commutation in Real Machine**
   - Armature Reaction
   - L di/dt Voltages
   - Solutions to the Problems with Commutation

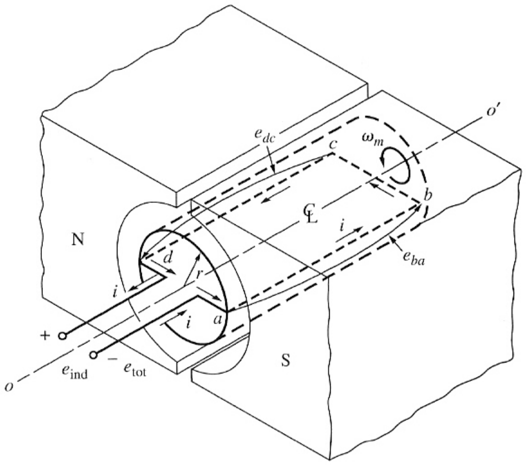
4. **The Internal Generated Voltage and Induced Torque Equations of Real DC Machine**

5. **The Construction of DC Machine**

6. **Power Flow and Losses in DC Machines**

1. A Simple Rotating Loop between Curved Pole Faces

The simplest rotating dc machine is shown below:



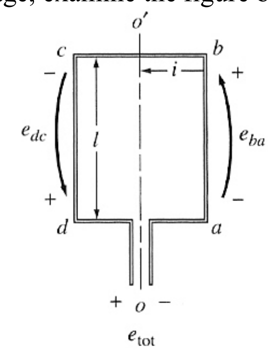
(a)

It consists of a single loop of wire rotating about a fixed axis. The rotating part is called rotor, and the stationary part is the stator.

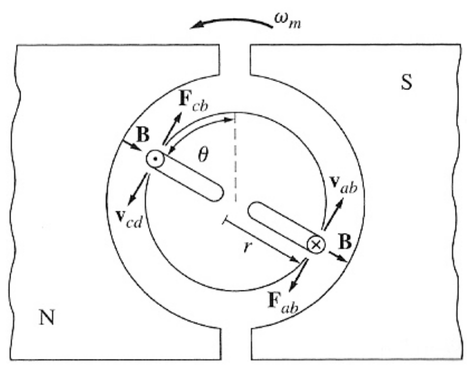
The magnetic field for the machine is supplied by the magnetic north and south poles. Since the air gap is of uniform width, the reluctance is the same everywhere under the pole faces.

The Voltage Induced in a Rotating Loop

If the rotor is rotated, a voltage will be induced in the wire loop. To determine the magnitude and shape of the voltage, examine the figure below:



(c)

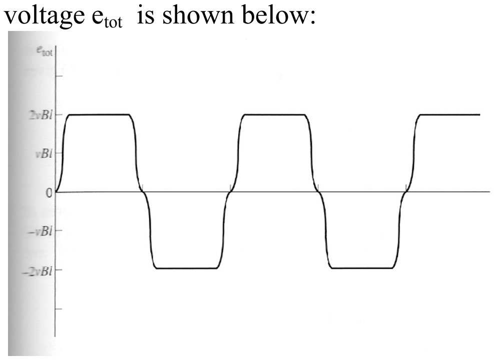


(d)

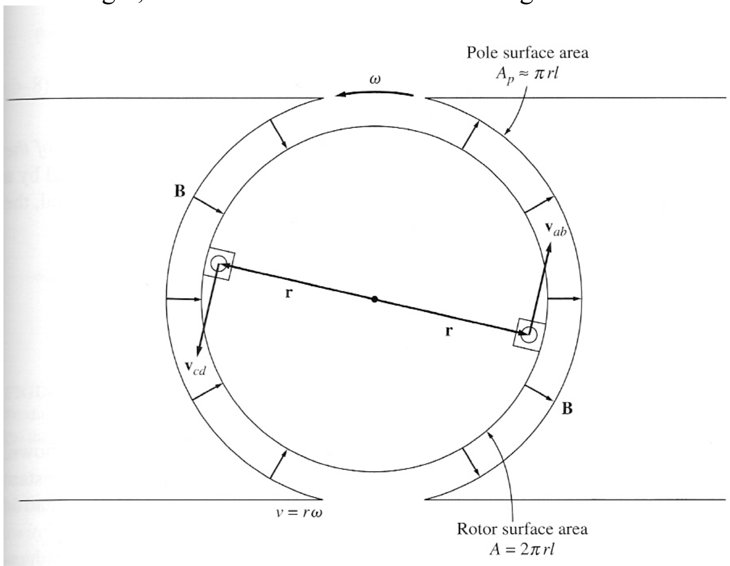
To determine the total voltage e_{tot} on the loop, examine each segment of the loop separately and sum all the resulting voltages. The voltage on each segment is given by $e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}$

Thus, the total induced voltage on the loop is: $e_{ind} = 2vBl$

When the loop rotates through 180°, segment *ab* is under the north pole face instead of the south pole face. At that time, the direction of the voltage on the segment reverses, but its magnitude remains constant. The resulting voltage e_{tot} is shown below:



There is an alternative way to express the e_{ind} equation, which clearly relates the behaviour of the single loop to the behaviour of larger, real dc machines. Examine the figure below:



The tangential velocity v of the edges of the loop can be expressed as v = rω. Substituting this expressing into the e_{ind} equation before gives:

$$e_{ind} = 2r\omega Bl$$

The rotor surface is a cylinder, so the area of the rotor surface A is equal to 2πrl. Since there are 2 poles, the area under each pole is $A_p = \pi rl$. Thus,

$$e_{ind} = \frac{2}{\pi} A_P B \omega$$

Since the flux density B is constant everywhere in the air gap under the pole faces, the total flux under each pole is $\phi = A_P B$. Thus, the final form of the voltage equation is:

$$e_{ind} = \frac{2}{\pi} \phi \omega$$

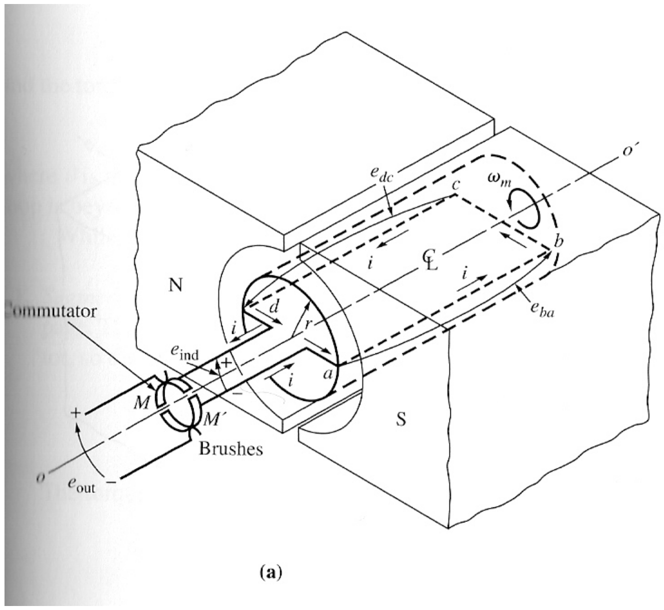
In general, the voltage in any real machine will depend on the same 3 factors:

1. the flux in the machine
2. The speed of rotation
3. A constant representing the construction of the machine.

Getting DC voltage out of the Rotating Loop

The voltage out of the loop is alternately a constant positive and a constant negative value. How can this machine be made to produce a dc voltage instead of the ac voltage?

This can be done by using a mechanism called commutator and brushes, as shown below:



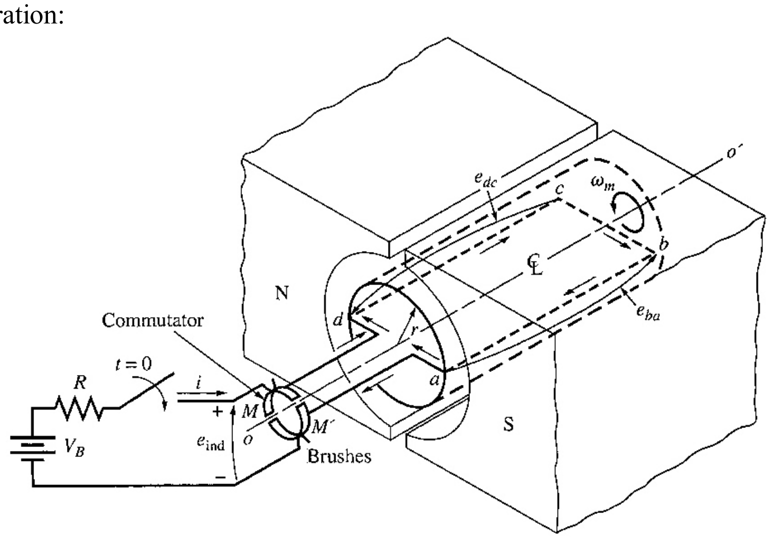
(a)

Here 2 semicircular conducting segments are added to the end of the loop, and 2 fixed contacts are set up at an angle such that at the instant when the voltage in the loop is zero, the contacts short-circuit the two segments.

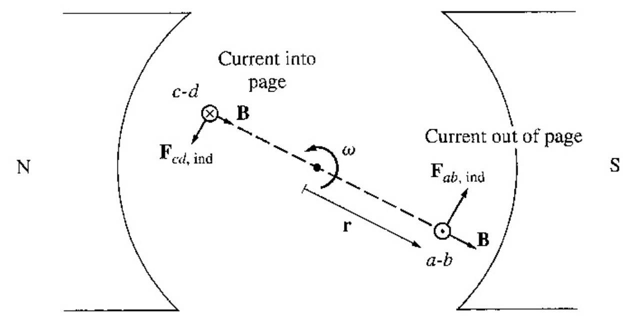
Thus, every time the voltage of the loop switches direction, the contacts also switches connections, and the output of the contacts is always built up in the same way. This connection-switching process is known as commutation. The rotating semicircular segments are called commutator segments, and the fixed contacts are called brushes.

The Induced Torque in the Rotating Loop

Suppose a battery is now connected to the machine as shown here, together with the resulting configuration:



(a)



(b)

How much torque will be produced in the loop when the switch is closed? The approach to take is to examine one segment of the loop at a time and then sum the effects of all the individual segments. The force on a segment of the loop is given by : $\mathbf{F} = i\,(\mathbf{l} \times \mathbf{B})$, and the torque on the segment is $\tau = r F \sin\theta$.

The resulting total induced torque in the loop is:

$$\tau_{ind} = 2\, rilB$$

By using the fact that $A_P = \pi rl$ and $\phi = A_P B$, the torque expression can be reduced to:

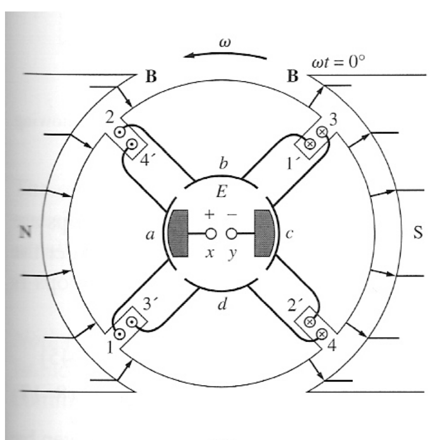
$$\tau_{ind} = \frac{2}{\pi}\phi i$$

In general, the torque in any real machine will depend on the same 3 factors:

1. The flux in the machine
2. The current in the machine
3. A constant representing the construction of the machine.

2. <u>Commutation in a Simple Four-Loop DC Machine</u>

Commutation is the process of converting the ac voltages and currents in the rotor of a dc machine to dc voltages and currents at its terminals. A simple 4 loop, 2 pole dc machine is shown here:

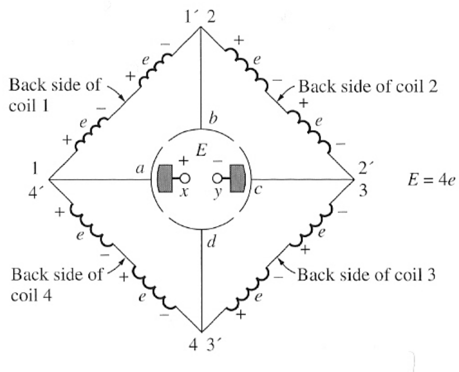


(a)

at ωt=0°

This machine has 4 complete loops buried in slots carved in the laminated steel of its rotor. The pole faces of the machine are curved to provide a uniform air-gap width and to give a uniform flux density everywhere under the faces.

The 4 loops of this machine are laid into the slots in a special manner. The "unprimed" end of each loop is the outermost wire in each slot, while the "primed" end of each loop is the innermost wire in the slot directly opposite. The winding's connections to the machine's commutator are shown below:



(b)

Notice that loop 1 stretches between commutator segments a and b, loop 2 stretches between segments b and c, and so forth around the rotor.

At the instant shown in figure (a), the 1, 2, 3' and 4' ends of the loops are under the north pole face, while the 1', 2', 3 and 4 ends of the loops are under the south pole face.

The voltage in each of the 1, 2, 3' and 4' ends of the loops is given by:

$$e_{ind} = (v \times B)\, l$$

$$e_{ind} = vBl \quad \text{(positive out of page)}$$

The voltage in each of the 1', 2', 3 and 4 ends of the loops is given by:

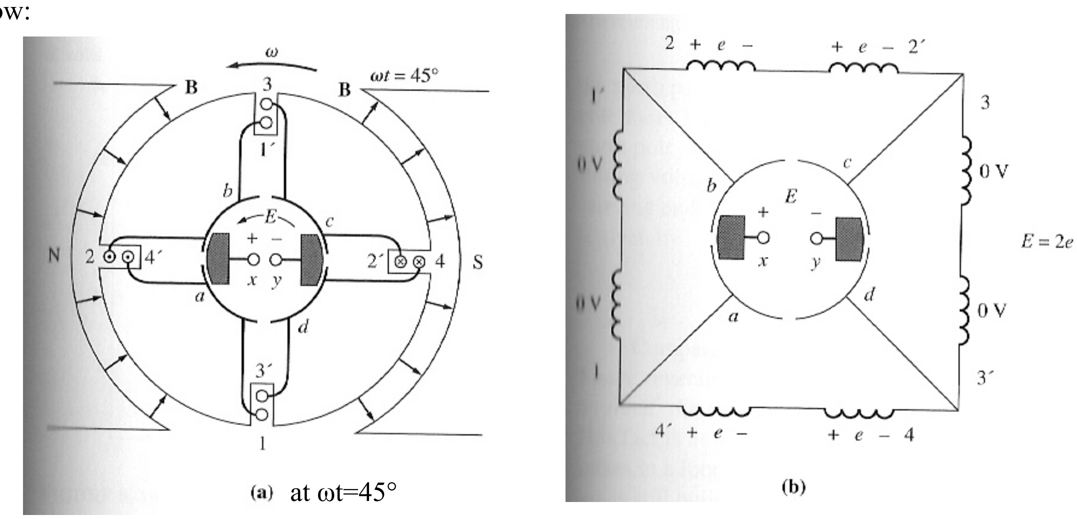
$$e_{ind} = (v \times B)\, l$$

$$e_{ind} = vBl \quad \text{(positive into the page)}$$

The overall result is shown in figure (b). Each coil represents one side (or conductor) of a loop. If the induced voltage on any one side of a loop is called e=vBl, then the total voltage at the brushes of the machine is $E = 4e \quad (\omega t=0°)$

Notice that there are two parallel paths for current through the machine. The existence of two or more parallel paths for rotor current is a common feature of all commutation schemes.

What happens to the voltage E of the terminals as the rotor continues to rotate? Examine the figures below:



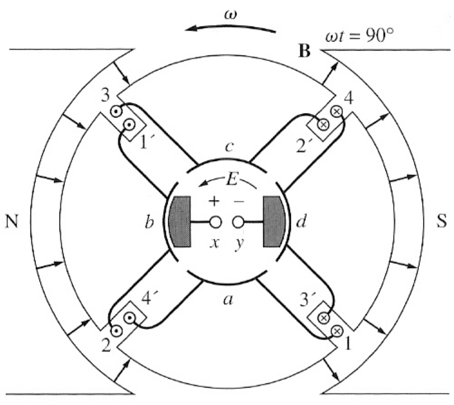
(a) at ωt=45°

(b)

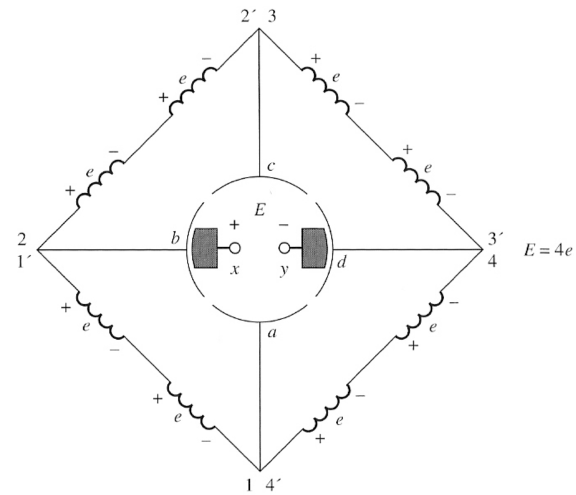
This figure shows the machine at time ωt=45°. At that time, loops 1 and 3 have rotated into the gap between the poles, so the voltage across each of them is zero. Notice that at this instant the brushes of the machine are shorting out commutator segments ab and cd. This happens just at the time when the loops between these segments have 0V across them, so shorting out the segments creates no problem. At this time, only loops 2 and 4 are under the pole faces, so the terminal voltage E is given by:

$$E = 2e \quad (\omega t = 45°)$$

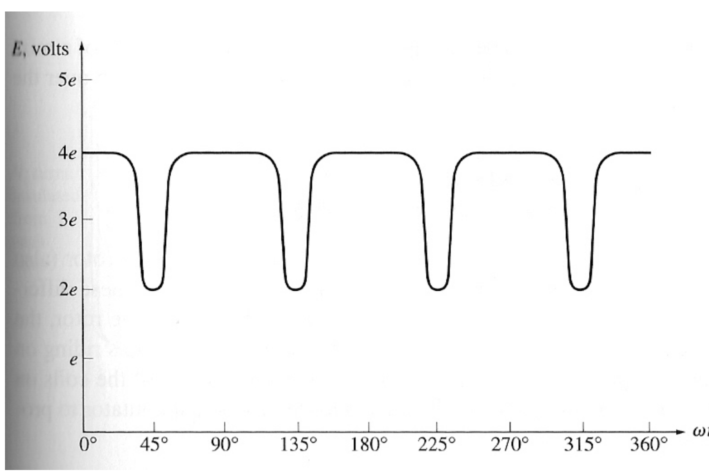
Now, let the rotor continue to turn another 45°. The resulting situation is shown below:



at ωt=90°



Here, the 1', 2, 3, and 4' ends of the loops are under the north pole face, and the 1, 2', 3' and 4 ends of the loops are under the south pole face. The voltages are still built up out of the page for the ends under the north pole face and into the page for the ends under the south pole face. The resulting voltage diagram is shown here:



There are now 4 voltage-carrying ends in each parallel path through the machine, so the terminal voltage E is given by:

$$E = 4e \quad (\omega t = 90°)$$

Notice that the voltages on loops 1 and 3 have reversed between the 2 pictures (from ωt=0° to ωt=90°), but since their connections have also reversed, the total voltage is still being built up in the same direction as before. This is the heart of every commutation scheme.

3. Problems with Commutation in Real Machine

In practice, there are two major effects that disturb the commutation process:

1. Armature Reaction
2. L di/dt voltages

Armature Reaction

If the magnetic field windings of a dc machine are connected to a power supply and the rotor of the machine is turned by an external source of mechanical power, then a voltage will be induced in the conductors of the rotor. This voltage will be rectified into dc output by the action of the machine's commutator.

Now, connect a load to the terminals of the machine, and a current will flow in its armature windings. This current flow will produce a magnetic field of its own, which will distort the original magnetic field from the machine's poles. This distortion of the flux in a machine as the load is increased is called *armature reaction*. It causes 2 serious problems in real dc machine.

Problem 1 : Neutral-Plane Shift

The magnetic neutral plane is defined as the plane within the machine where the velocity of the rotor wires is exactly parallel to the magnetic flux lines, so that e_{ind} in the conductors in the plane is exactly zero.

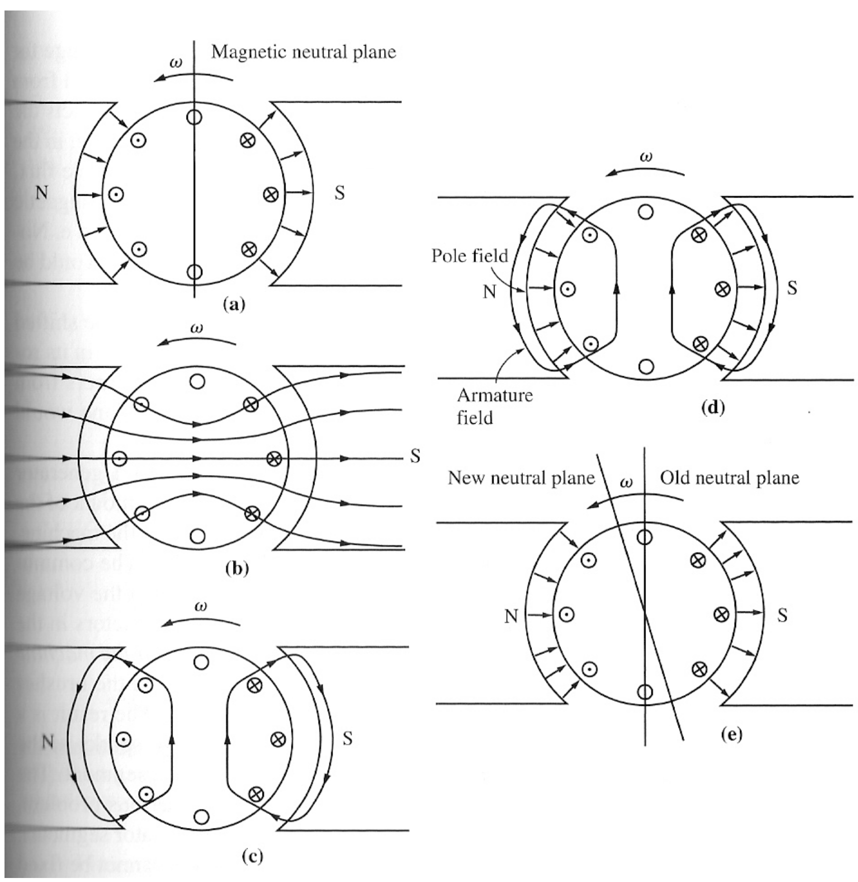


Figure (a) shows a two poles machine. Notice that the flux is distributed uniformly under the pole faces. The rotor windings shown have voltages built up out of the page for wires under the north pole and into the page for wires under the south pole face. The neutral plane in this machine is exactly vertical.

Now, suppose a load is connected to this machine so that it acts as a generator. Current will flow out of the positive terminal of the generator, so current will be flowing out of the page for wires under the north pole face and into the page for wires under the south pole face. This current flow produces a magnetic field from the rotor windings, figure (c).

This rotor magnetic field affects the original magnetic field from the poles that produced the generator's voltage in the first place. In some places under the pole surfaces, it subtracts from the pole flux, and in other places it adds to the pole flux. The overall result is that the magnetic flux in the air gap of the machine is skewed, figure (d) and (e). Notice that the place on the rotor where the induced voltage in a conductor would be zero (the neutral plane) has shifted.

For the generator shown here, the magnetic neutral plane shifted in the direction of rotation. If this machine had been a motor, the current in its rotor would be reversed and the flux would bunch up in the opposite corners from the bunches shown in the figure. As a result, the magnetic neutral plane would shift the other way.

In general, the neutral-plane shifts in the direction of motion for generator and opposite to the direction of motion for a motor. Furthermore, the amount of the shift depends on the amount of rotor current and hence on the load of the machine.

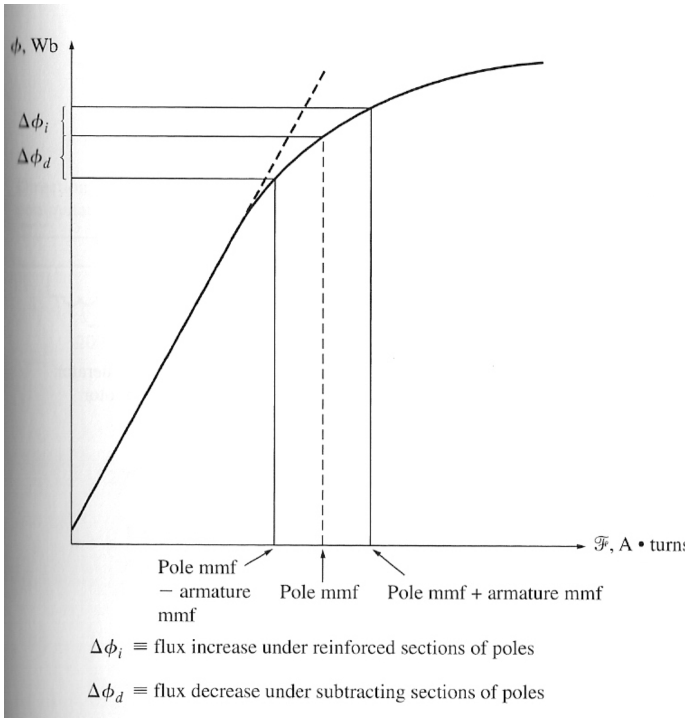
What is so important regarding the neutral-plane shift?

The commutator must short out commutator segments just at the moment when the voltage across them is equal to zero. If the brushes are set to short out conductors in the vertical plane, then the voltage between segments is indeed zero until the machine is loaded. When the machine is loaded, the neutral-plane shifts, and the brushes short out commutator segments with a finite voltage across them. The result is a current flow circulating between the shorted segments and large sparks at the brushes when the current path is interrupted as the brush leaves a segment. The end result is arcing and sparking at the brushes. This is a very serious problem, since it leads to drastically reduced brush life, pitting of the commutator segments, and higher maintenance costs. Notice that this problem cannot be fixed even by placing the brushes over the full-load neutral plane, because then they would spark at no load.

In extreme cases, the neutral-plane shift can even lead to flashover in the commutator segments near the brushes. The air near the brushes in a machine is normally ionized as a result of the sparking on the brushes. Flashover occurs when the voltage of adjacent commutator segments gets large enough to sustain an arc in the ionized air above them. If flashover occurs, the resulting arc can even melt the commutator's surface.

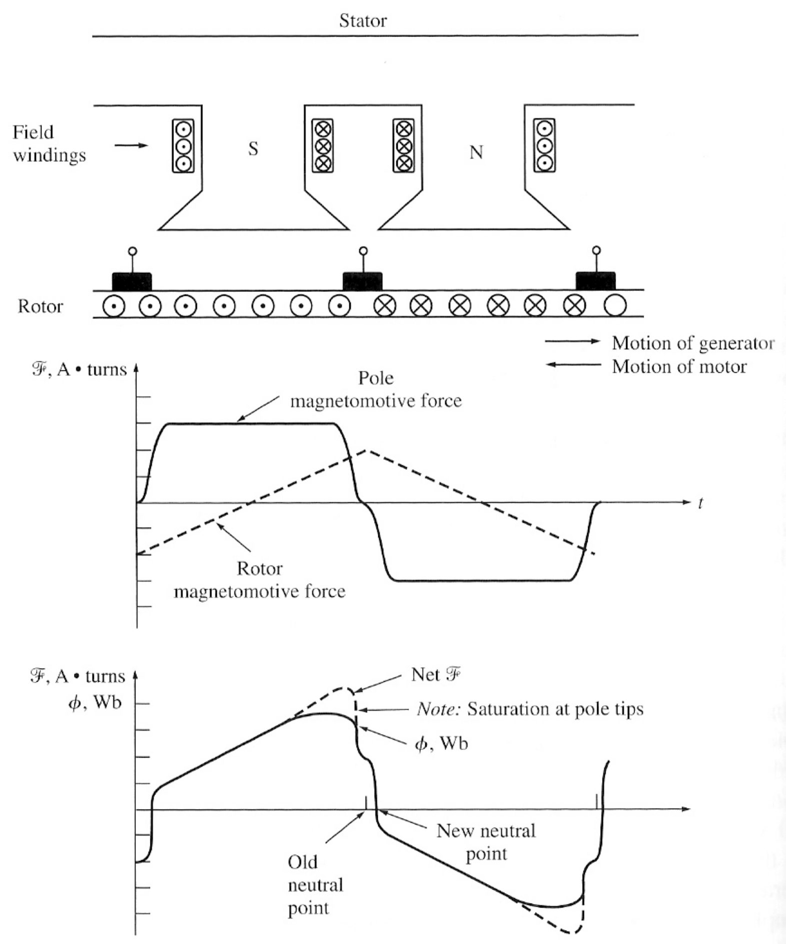
Problem 2 : Flux Weakening

Refer to the magnetization curve below:



Most machine operate at flux densities near the saturation point. Therefore, at locations on the pole surfaces where the rotor mmf adds to the pole mmf, only a small increase in flux occurs. But at locations on the pole surfaces where the rotor mmf subtracts from the pole mmf, there is a larger decrease in flux. The net result is that the total average flux under the entire pole face is decreased.

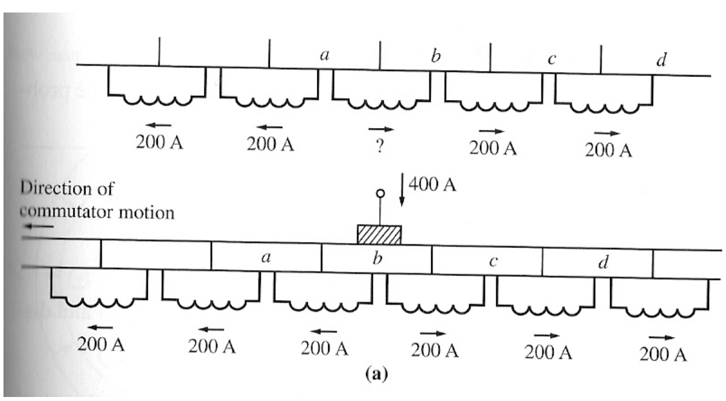
The flux weakening causes problems in both generators and motors. In generators, the effect of flux weakening is simply to reduce the voltage supplied by the generator for any given load. In motors, the effect can be more serious. When the flux in a motor is decreased, its speed increases. But increasing the speed of a motor can increase its load, resulting in more flux weakening.



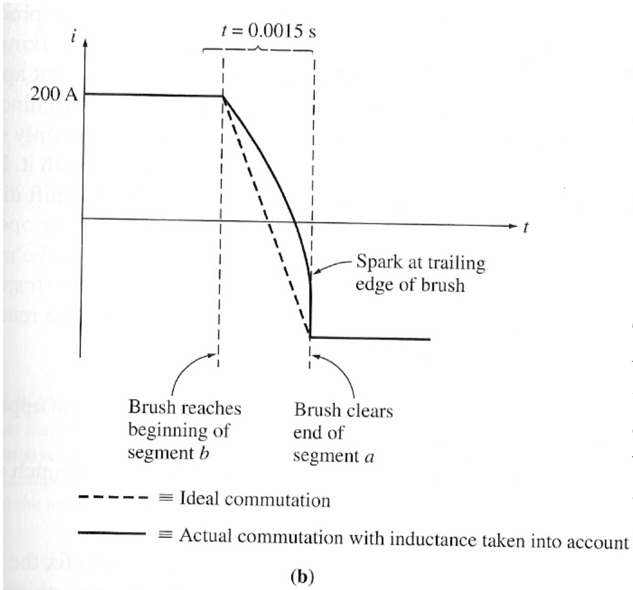
The flux and mmf under the pole faces in a dc machine. At those points where the mmf subtract, the flux closely follows the net mmf in the iron; but at those points where the mmf add, saturation limits the total flux present. Note also the neutral point of the rotor has shifted.

L di/dt Voltages

The second major problem is the L di/dt voltage that occurs in commutator segments being shorted out by the brushes, sometimes called inductive kick.



The reversal of current flow in a coil undergoing commutation. Note that the current in the coil between segments a and b must reverse direction while the brush shorts together the two commutator segments.



The current reversal in the coil undergoing commutation as a function of time for both ideal commutation and real commutation, with the coil inductance taken into account.

These figures represents a series of commutator segments and the conductors connected between them. Assuming that the current in the brush is 400A, the current in each path is 200A. Notice that when a commutator segment is shorted out, the current flow through that commutator segment must reverse. How fast must this reversal occur?

Assumming that the machine is turning at 800r/min and that there are 50 commutator segments, each commutator segment moves under a brush and clears it again in t=0.0015s. Therefore, the rate of change in current with respect to time in the shorted loop must average

$$\frac{di}{dt} = \frac{400A}{0.0015s} = 266667A/s$$

With even a tiny inductance in the loop, a very significant inductive voltage kick v = L di/dt will be induced in the shorted commutator segment. This high voltage naturally causes sparking at the brushes of the machine, resulting in the same arcing problems that the neutral-plane shift causes.

Solutions to the Problems with Commutation

Three approaches have been developed to partially or completely correct the problems of armature reaction and L di/dt voltages:

1. Brush Shifting
2. Commutation poles or interpoles
3. Compensating windings

Brush Shifting

The first attempts to improve the process of commutation in real dc machines started with attempts to stop the sparking at the brushes caused by the neutral-plane shifts and L di/dt effects.

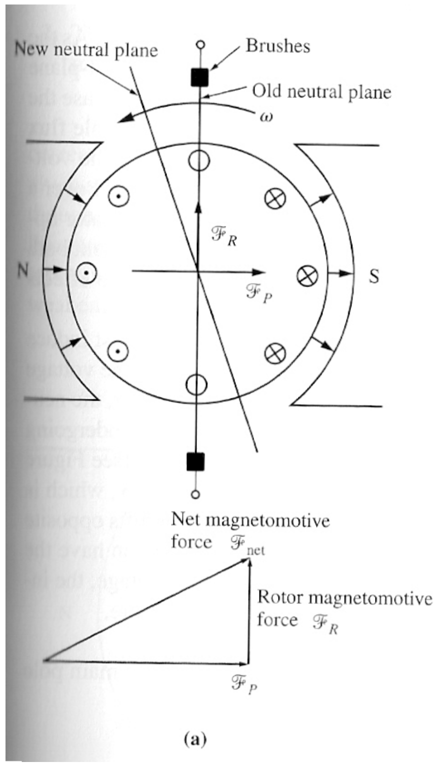
The first approach taken by machine designers was simple: If the neutral plane of the machine shifts, why not shift the brushes with it in order to stop the sparking? This method is good but there are several problems:

- The neutral plane moves with every change in load, and the shift direction reverses when the machine goes from motor operation to generator operation. Therefore, someone has to adjust the brushes every time the load changed. Although this method may have stopped the brush sparking, it actually aggravated the flux-weakening effect of the armature reaction in the machine.

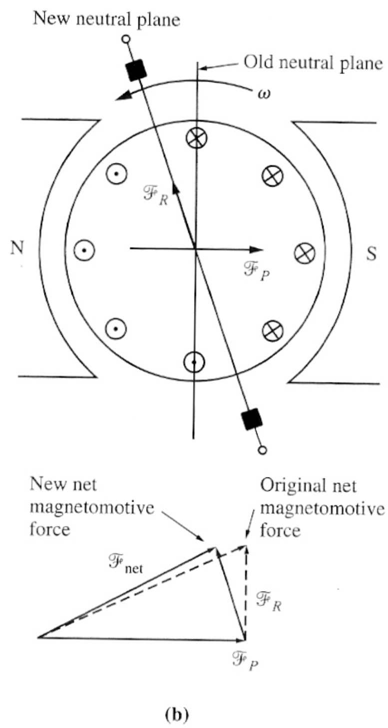
This is true because of 2 effects:

i) The rotor mmf now has a vector component that opposes the mmf from the poles.

ii) The change in armature current distribution causes the flux to bunch up even more at the saturated parts of the pole faces.



(a) The net mmf in a dc machine with its brushes in the vertical plane.



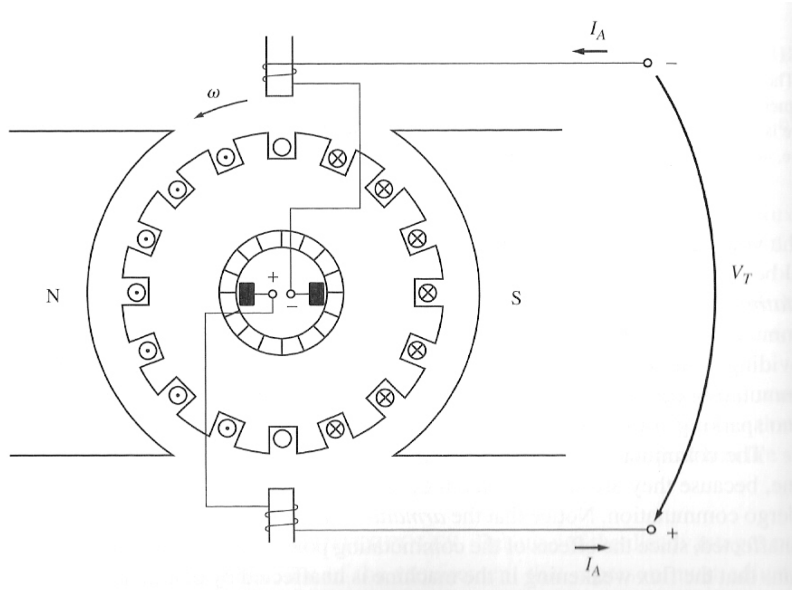
(b) The net mmf in a dc machine with its brushes over the shifted neutral plane. Notice that now there is a component of armature mmf directly opposing the poles' mmf and the net mmf in the machine is reduced.

<u>Commutation Poles or Interpoles</u>

The basic idea here is that if the voltage in the wires undergoing commutation can be made zero, then there will be no sparking at the brushes. To accomplish this, small poles, called commutating poles or interpoles, are placed midway between the main poles. These commutating poles are located directly over the conductors being commutated. By providing a flux from the commutating poles, the voltage in the coils undergoing commutation can be exactly cancelled. If the cancellation is exact, then there will be no sparking at the brushes.

The commutating poles do not otherwise change the operation of the machine, because they are so small that they affect only the few conductors about to undergo commutation. Notice that the armature reaction under the main pole faces is unaffected, since the effects of the commutating poles do not extend that far. This means that the flux weakening in the machine is unaffected by commutating poles.

How is cancellation of the voltage in the commutator segments accomplished for all values of load? This is done by simply connecting the interpole windings in series with the windings on the rotor, as shown below:

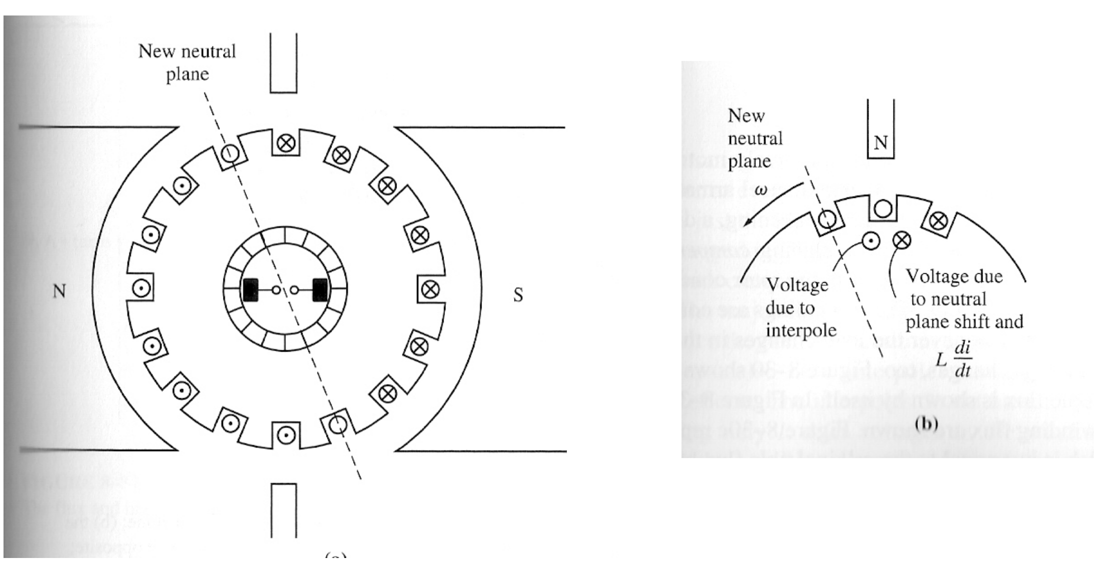


As the load increases, and the rotor current increases, the magnitude of the neutral-plane shift and the size of the L di/dt effects increase too. Both these effects increase the voltage in the conductors undergoing commutation.

However, the interpole flux increases too, producing a larger voltage in the conductors that opposes the voltage due to the neutral-plane shift. The net result is that their effects cancel over a broad range of loads. Note that interpoles work for both motor and generator operation, since when the machine changes from motor to generator, the current both in its rotor and in its interpoles reverses direction. Therefore, the voltage effects from them still cancel.

What polarity must the flux in the interpoles be?

The interpoles must induce a voltage in the conductors undergoing commutation that is opposite to the voltage caused by neutral-plane shift and L di/dt effects. In the case of a generator, the neutral plane shifts in the direction of rotation, meaning that the conductors undergoing commutation have the same polarity of voltage as the pole they just left.



Determining the required polarity of an interpole. The flux from the interpole must produce a voltage that opposes the existing voltage in the conductor.
To oppose this voltage, the interpoles must have the opposite flux, which is the flux of the upcoming pole. In a motor, however, the neutral plane shifts opposite to the direction of rotation, and the conductors undergoing commutation have the same flux as the pole they are approaching. In order to oppose this voltage, the interpoles must have the same polarity as the previous main pole. Therefore,

1. The interpoles must be of the same polarity as the next upcoming main pole in a generator.
2. The interpoles must be of the same polarity as the previous main pole in a motor.

The use of commutating poles or interpoles is very common, because they correct the sparking problems of dc machines at a fairly low cost. They are almost always found in any dc machine of 1HP or larger.

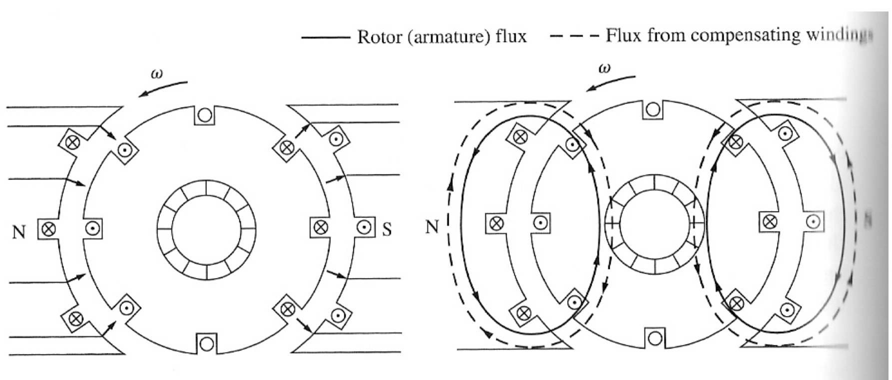
It is important to realize that they do nothing for the flux distribution under the pole faces, so the flux-weakening problem is still present. Most medium-size, general-purpose motors correct for sparking problems with interpoles and just live with the flux-weakening effects.

Compensating Windings

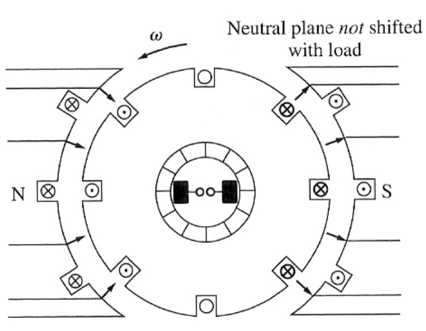
For very heavy, severe duty cycle motors, the flux-weakening problem can be very serious. To completely cancel armature reaction and thus eliminate both neutral-plane shift and flux weakening, a different technique was developed.

This technique involves placing compensating windings in slots carved in the faces of the poles parallel to the rotor conductors, to cancel the distorting effect of armature reaction. These windings are connected in series with the rotor windings, so that whenever the load changes in the rotor, the current in the compensating windings changes too.

The figures below show the basic concept:



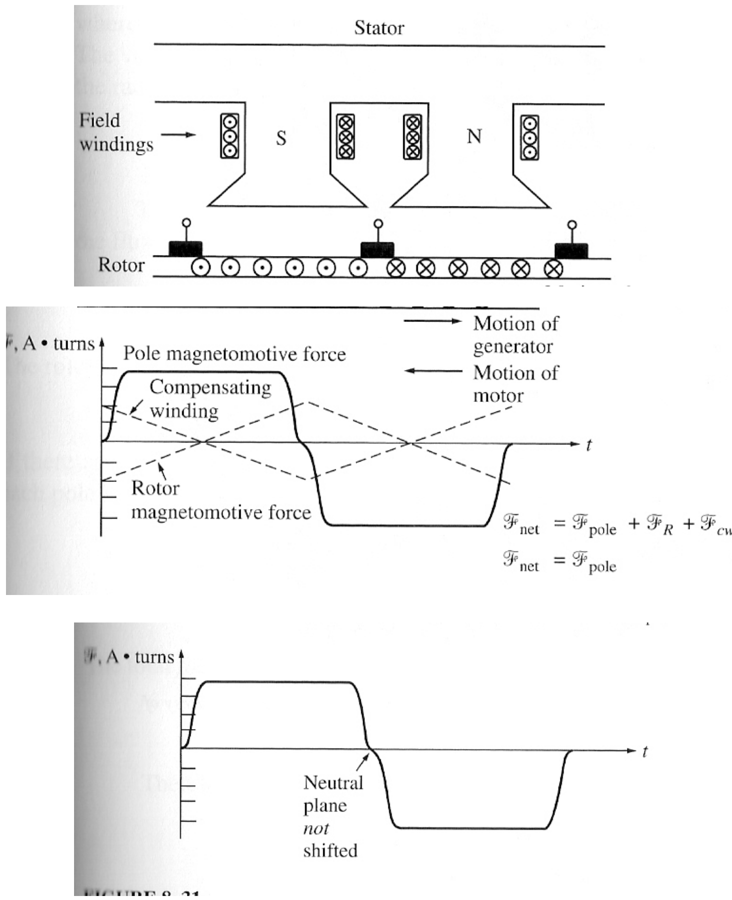
(a) (b)



(c)

The effect of compensating windings in a dc machine:
(a) The pole flux in the machine
(b) The fluxes from the armature and compensating windings (notice that they are equal and opposite)
(c) The net flux in the machine, which is just the original flux.

Another careful development of the effect of compensating windings on a dc machine is illustrated below:



Notice that the mmf due to the compensating windings is equal and opposite to the mmf due to the rotor at every point under the pole faces. The resulting net mmf is just the mmf due to the poles, so the flux in the machine is unchanged regardless of the load on the machine.

The major disadvantage of compensating windings is that they are expensive, since they must be machined into the faces of the poles. Any motor that uses them must also have interpoles, since compensating windings do not cancel L di/dt effects. The interpoles do not have to be as strong, though since they are cancelling only L di/dt voltages in the windings, and not the voltages due to neutral-plane shifting. Because of the expense of having both compensating windings and interpoles on such a machine, these windings are used only where the extremely severe nature of a motor's duty demands them.

4. <u>The Internal Generated Voltage and Induced Torque Equations of Real DC Machine</u>

How can the voltage in the rotor windings of a real dc machine be determined?

The voltage out of the armature of a real dc machine is equal to the number of conductors per current path times the voltage on each conductor.

The voltage in any single conductor under the pole faces was previously shown to be $e_{ind} = e = vBl$

The voltage out of the armature of a real machine is thus:

$$E_A = \frac{ZvBl}{a}$$

Where Z is the total number of conductors and a is the number of current paths. The velocity of each conductor in the rotor is v=rω so,

$$E_A = \frac{Zr\omega Bl}{a}$$

And with $\phi = BA_P$ and $A = 2\pi rl$, and if there are P poles on the machine, then the total area A is

$$A_P = \frac{A}{P} = \frac{2\pi rl}{P}$$

The total flux per pole in the machine:

$$\phi = BA_P = \frac{B(2\pi rl)}{P} = \frac{2\pi rlB}{P}$$

Therefore, the internal generated voltage in the machine can be expressed as:

$$\begin{aligned} E_A &= \frac{Zr\omega Bl}{a} \\ &= \left(\frac{ZP}{2\pi a}\right)\left(\frac{2\pi rlB}{P}\right)\omega \\ E_A &= \frac{ZP}{2\pi a}\phi\omega \\ E_A &= K\phi\omega \end{aligned}$$

How much torque is induced in the armature of a real dc machine?

The torque on the armature of a real machine is equal to the number of conductors Z times the torque on each conductor. The torque in any single conductor under the pole faces was previously shown to be

$\tau_{cond} = r\, I_{cond}\, lB$

If there are a current paths in the machine, then the total armature current I_A is split among the a current paths, so the current in a single conductor is given by

$$I_{cond} = \frac{I_A}{a}$$

And the torque in a single conductor on the motor is:

$$\tau_{cond} = \frac{rI_A lB}{a}$$

Since there are Z conductors, the total induced torque in a dc machine rotor is:

$$\tau_{ind} = \frac{ZrlBI_A}{a}$$

The flux per pole in this machine:

$$\phi = BA_P = \frac{B(2\pi rl)}{P} = \frac{2\pi rlB}{P}$$

So the total induced torque is:

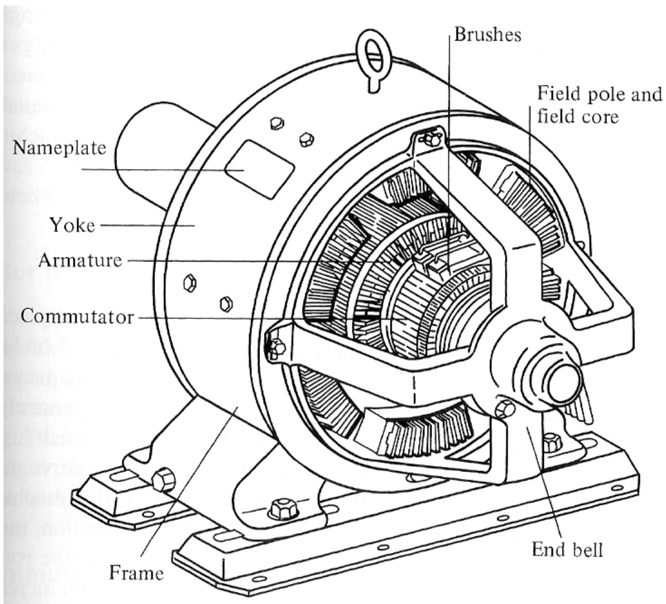
$$\tau_{ind} = \frac{ZP}{2\pi a}\phi I_A$$

Finally,

$$\tau_{ind} = K\phi I_A$$

5. <u>The Construction of DC Machine</u>

A simplified diagram of a dc machine:



The physical structure of the machine consists of 2 parts: the stator and the rotor.

The stationary part consists of the frame, and the pole pieces, which project inward and provide a path for the magnetic flux. The ends of the pole pieces that are near the rotor spread out over the rotor surface to distribute its flux evenly over the rotor surface. These ends are called the pole shoes. The exposed surface of a pole shoe is called a pole face, and the distance between the pole face and the rotor is the air gap.

Two principal windings on a dc machine:

i- the armature windings: the windings in which a voltage is induced (rotor)

ii- the field windings: the windings that produce the main magnetic flux (stator)

Note: because the armature winding is located on the rotor, a dc machine's rotor is sometimes called an armature.

6. Power Flow and Losses in DC Machines

$$\text{Efficiency} = \frac{P_{out} - P_{loss}}{P_{in}} \times 100\%$$

The Losses in DC Machine:

1. Electrical or Copper Losses (I^2R Loss)

Armature loss: $P_A = I_A^2 R_A$
Field loss: $P_F = I_F^2 R_F$

2. Brush Losses

$$P_{BD} = V_{BD}^2 I_A$$

3. Core Losses

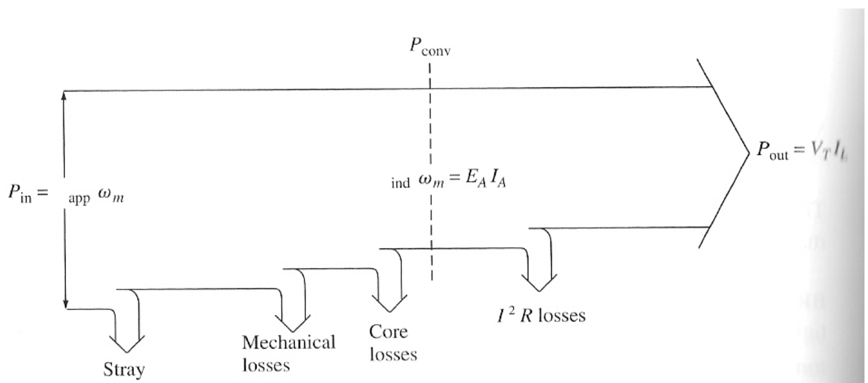
- Hysteresis and Eddy Current Loss

4. Mechanical Losses

- Friction and windage loss

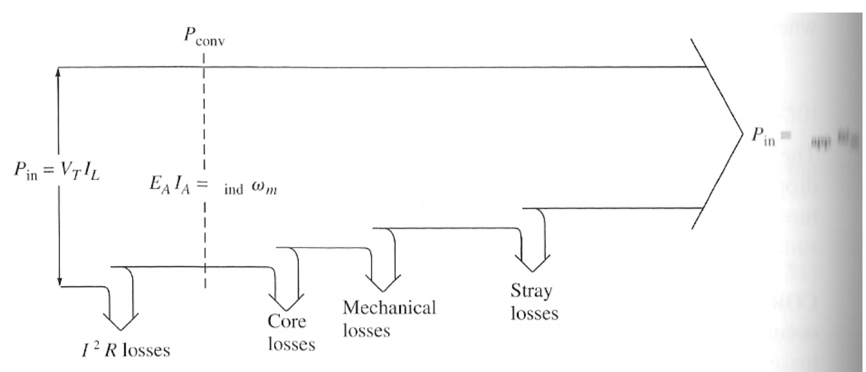
5. Stray Loss

The Power-Flow Diagram:



(a)

Generator



(b)

Motor

CHAPTER 9 – DC MOTORS

Summary:

1. **The Equivalent Circuit of a DC Motor**

2. **The Magnetization Curve of a DC Machine**

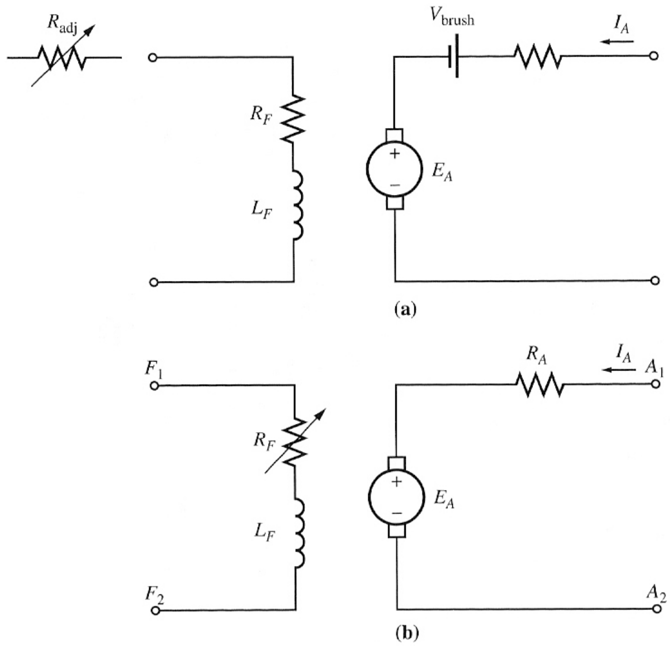
3. **Separately Excited and Shunt DC Motors**
   - The Terminal Characteristics of a Shunt DC Motor
   - Nonlinear Analysis of a Shunt DC Motor
   - Speed Control of Shunt DC Motors
   - The Effect of an Open Field Circuit

4. **The Permanent-Magnet DC Motor**

5. **The Series DC Motor**
   - Induced Torque in a Series DC Motor
   - The Terminal Characteristic of a Series DC Motor
   - Speed Control of Series DC Motors.

6. **The Compounded DC Motor**
   - The Torque-Speed Characteristic of a Cumulatively Compounded DC Motor
   - The Torque-Speed Characteristic of a Differentially Compounded DC Motor
   - The Nonlinear analysis of Compounded DC Motors
   - Speed Control in the Cumulatively Compounded DC Motor

1. The Equivalent Circuit of a DC Motor



(a) The equivalent circuit

(b) A simplified equivalent circuit eliminating the brush voltage drop and combining R_{adj} with the field resistance.

In this figure, the armature circuit is represented by an ideal voltage source E_A and a resistor R_A. This representation is really the Thevenin equivalent of the entire rotor structure, including rotor coils, interpoles and compensating windings, if present.

The brush voltage drop is represented by a small battery V_{brush} opposing the direction of current flow in the machine.

The field coils, which produce the magnetic flux in the motor are represented by inductor L_F and resistor R_F. The separate resistor R_{adj} represents an external variable resistor used to control the amount of current in the field circuit.

Some of the few variations and simplifications:

i- The brush drop voltage is often only a very tiny fraction of the generated voltage in the machine. Thus, in cases where it is not too critical, the brush drop voltage may be left out or included in the R_A.

ii- The internal resistance of the field coils is sometimes lumped together with the variable resistor and the total is called R_F.

iii- Some generators have more than one field coil, all of which appear on the equivalent circuit.

The internal generated voltage is given by:

$$E_A = K\phi\omega$$

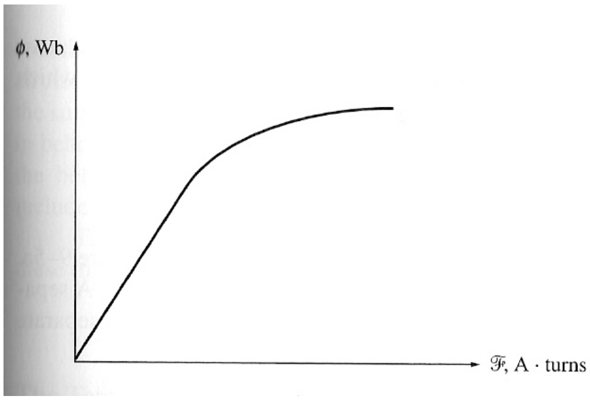
and the torque induced is

$$\tau_{ind} = K\phi I_A$$

2. The Magnetization Curve of a DC Machine

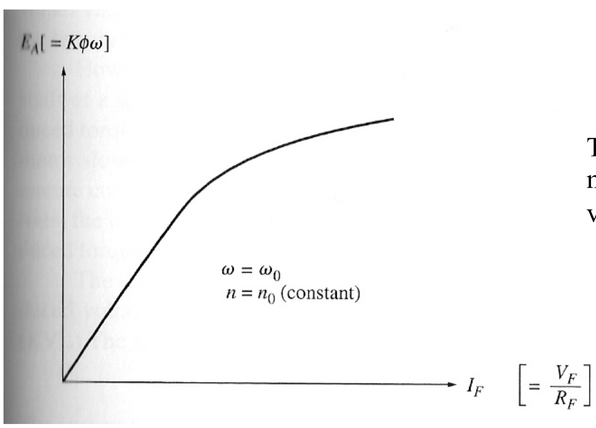
E_A is directly proportional to flux and the speed of rotation of the machine. How is the E_A related to the field current in the machine?

The field current in a dc machine produces a field mmf given by $F=N_FI_F$. This mmf produces a flux in the machine in accordance with its magnetization curve, shown below:



The magnetization curve of a ferromagnetic material

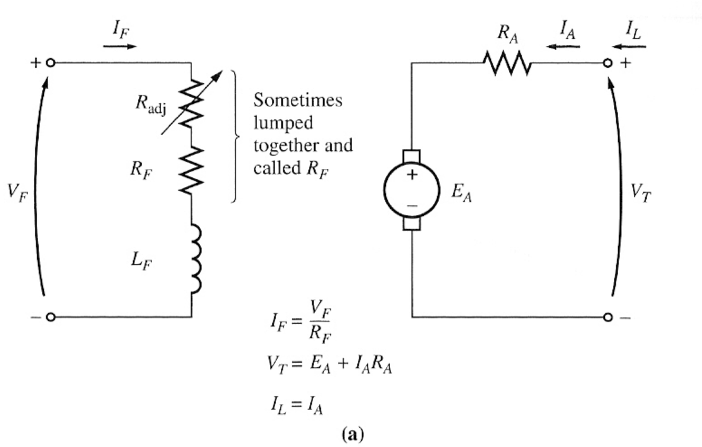
Since the field current is directly proportional to the mmf and since E_A is directly proportional to flux, it is customary to present the magnetization curve as a plot of E_A vs field current for a given speed ω_o.



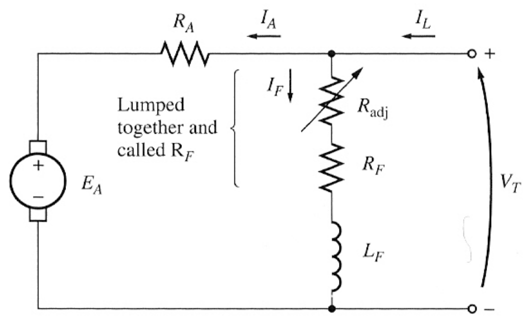
The magnetization curve of a dc machine expressed as a plot of E_A versus I_F for a fixed speed ω_o.

NOTE: Most machines are designed to operate near the saturation point on the magnetization curve. This implies that a fairly large increase in field current is often necessary to get a small increase in E_A when operation is near full load.

3. Separately Excited and Shunt DC Motors



The equivalent circuit of a separately excited dc motor



$$I_F = \frac{V_T}{R_F}$$

$$V_T = E_A + I_A R_A$$

$$I_L = I_A + I_F$$

(b)

The equivalent circuit of a shunt dc motor

A separately excited dc motor is a motor whose field circuit is supplied from a separate constant-voltage power supply, while a shunt dc motor is a motor whose field circuit gets its power directly across the armature terminals of the motor.

When the supply voltage to a motor is assumed constant, there is no practical difference in behaviour between these two machines. Unless otherwise specified, whenever the behaviour of a shunt motor is described, the separately excited motor is included too.

The KVL equation for the armature circuit is:

$$V_T = E_A + I_A R_A$$

The Terminal Characteristics of a Shunt DC Motor

A terminal characteristic of a machine is a plot of the machine's output quantities versus each other. For a motor, the output quantities are shaft torque and speed, so the terminal characteristic of a motor is a plot of its output torque versus speed.

How does a shunt dc motor respond to a load?

Suppose that the load on the shaft of a shunt motor is increased. Then the load torque τ_{load} will exceed the induced torque τ_{ind} in the machine, and the motor will start to slow down. When the motor slows down, its internal generated voltage drops ($E_A = K\phi\omega\downarrow$), so the armature current in the motor $I_A = (V_T - E_A\downarrow)/R_A$ increases. As the armature current rises, the induced torque in the motor increases ($\tau_{ind} = K\phi I_A\uparrow$), and finally the induced torque will equal the load torque at a lower mechanical speed of rotation.

The output characteristic of a shunt dc motor can be derived from the induced voltage and torque equations of the motor plus the KVL.

KVL → $V_T = E_A + I_A R_A$

The induced voltage $E_A = K\phi\omega$, so

$$V_T = K\phi\omega + I_A R_A$$

Since $\tau_{ind} = K\phi I_A$, current I_A can be expressed as:

$$I_A = \frac{\tau_{ind}}{K\phi}$$

Combining the V_T and I_A equations:

$$V_T = K\phi\omega + \frac{\tau_{ind}}{K\phi} R_A$$

Finally, solving for the motor's speed:

$$\omega = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2}\tau_{ind}$$

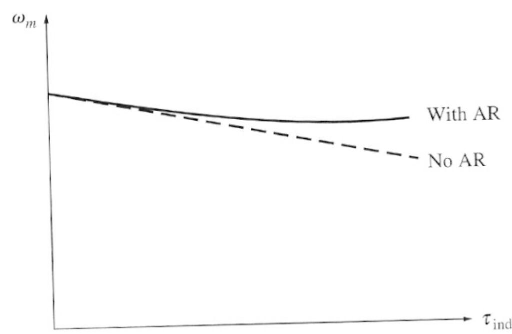
This equation is just a straight line with a negative slope. The resulting torque-speed characteristic of a shunt dc motor is shown here:

(a)

Torque-speed characteristic of a shunt or separately excited dc motor with compensating windings to eliminate armature reaction

It is important to realize that, in order for the speed of the motor to vary linearly with torque, the other terms in this expression must be constant as the load changes. The terminal voltage supplied by the dc power source is assumed to be constant - if it is not constant, then the voltage variations will affect the shape of the torque-speed curve.

Another effect internal to the motor that can also affect the shape of the torque-speed curve is armature reaction. If a motor has armature reaction, then as its load increases, the flux-weakening effects reduce its flux. From the motor speed equation above, the effect of reduction in flux is to increase the motor's speed at any given load over the speed it would run at without armature reaction. The torque-speed characteristic of a shunt motor with armature reaction is shown below:



(b)

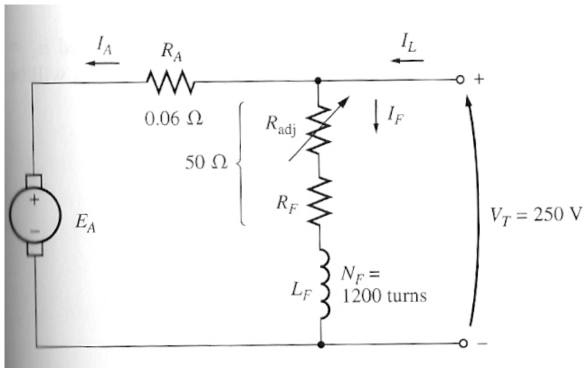
Torque-speed characteristic of the motor with armature reaction present.

If a motor has compensating windings, there will be no flux weakening problems and the flux in the motor will be constant.

If a shunt dc motor has compensating windings so that flux is constant regardless of load, and the motor's speed and armature current are known at any one value of load, then it is possible to calculate its speed at any other value of load, as long as the armature current at that load is known or can be determined.

Example 9.1

A 50HP, 250V, 1200 r/min DC shunt motor with compensating windings has an armature resistance (including the brushes, compensating windings, and interpoles) of 0.06 Ω. Its field circuit has a total resistance $R_{adj} + R_F$ of 50 Ω, which produces a no-load speed of 1200r/min. There are 1200 turns per pole on the shunt field winding (Figure below)



(a) Find the speed of this motor when its input current is 100A.
(b) Find the speed of this motor when its input current is 200A.
(c) Find the speed of this motor when its input current is 300A.

Nonlinear Analysis of a Shunt DC Motor

The flux and hence the internal generated voltage E_A of a dc machine is a non linear function of its magnetomotive force. Therefore, anything that changes the mmf in a machine will have a non linear effect on the E_A of the machine. Since the change in E_A cannot be calculated analytically, the magnetization curve of the machine must be used. Two principal contributors to the mmf in the machine are its field current and its armature reaction, if present.

Since the magnetization curve is a plot of E_A vs I_F for a given speed ω_o , the effect of changing a machine's field current can be determined directly from its magnetization curve.

If a machine has armature reaction, its flux will be reduced with each increase in load. The total mmf in a shunt dc motor is the field circuit mmf less the mmf due to armature reaction (AR):

$$F_{net} = N_F I_F - F_{AR}$$

Since the magnetization curves are expressed as plots of E_A vs field current, it is customary to define an equivalent field current that would produce the same output voltage as the combination of all the mmf in the machine. The resulting voltage E_A can then be determined by locating that equivalent field current on the magnetization curve. The equivalent field current:

$$I_F^* = I_F - \frac{F_{AR}}{N_F}$$

One other effect must be considered when non linear analysis is used to determine E_A of a dc motor. The magnetization curves for a machine are drawn for a particular speed, usually the rated speed of the machine. How can the effects of a given field current be determined if the motor is turning at other than rated speed?

The equation for the induced voltage in a dc machine when speed is expressed as rev/min: $E_A = K'\phi n$

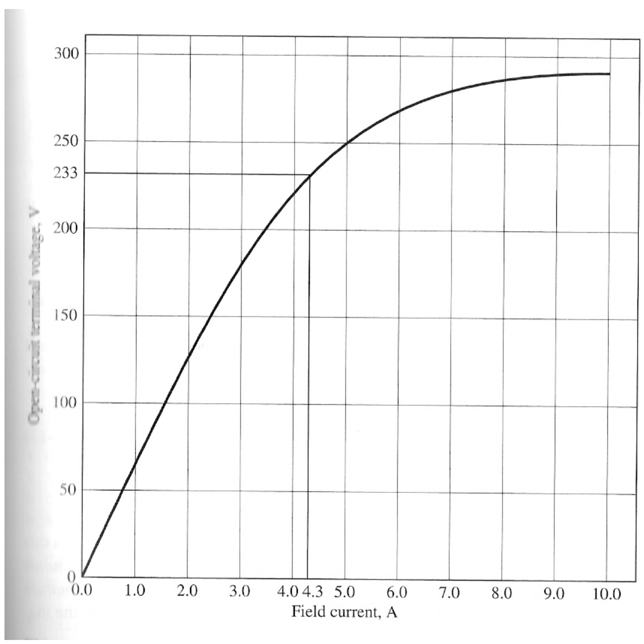
For a given effective field current, the flux in the machine is fixed, so the E_A is related to speed by:

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

where E_{A0} and n_0 represent the reference values of voltages and speed respectively. If the reference conditions are known from the magnetization curve and the actual E_A is known, then it is possible to determine the actual speed n.

<u>Example 9.2</u>

A 50HP, 250V, 1200r/min DC shunt motor without compensating windings has an armature resistance (including the brushes and interpoles) of 0.06 Ω. Its field circuit has a total resistance $R_{adj} + R_F$ of 50 Ω, which produces a no-load speed of 1200r/min. There are 1200 turns per pole on the shunt field winding, and the armature reaction produces a demagnetising magnetomotive force of 840 A turns at a load current of 200A. The magnetization curve of this machine is shown below:



(a) Find the speed of this motor when its input current is 200A.
(b) This motor is essentially identical to the one in Example 9.1 except for the absence of compensating windings. How does its speed compare to that of the previous motor at a load current of 200A?

Speed Control of Shunt DC Motors

Two common methods (as already been seen in Chapter 1 simple linear machine):

i- Adjusting the field resistance R_F (and thus the field flux)
ii- Adjusting the terminal voltage applied to the armature.

Less common method:

iii- Inserting a resistor in series with the armature circuit.

Changing the Field Resistance

If the field resistance increases, then the field current decreases ($I_F\downarrow = V_T/R_F\uparrow$), and as the field current decreases, the flux decreases as well. A decrease in flux causes an instantaneous decrease in the internal generated voltage $E_A\downarrow$ ($=K\phi\downarrow\omega$), which causes a large increase in the machine's armature current since,

$$I_A \uparrow = \frac{V_T - E_A \downarrow}{R_A}$$

The induced torque in a motor is given by $\tau_{ind} = K\phi I_A$. Since the flux in this machine decreases while the current I_A increases, which way does the induced torque change?

Look at this example:

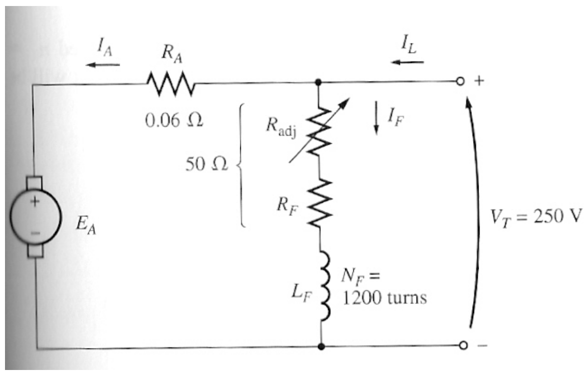


Figure above shows a shunt dc motor with an internal resistance of 0.25Ω. It is currently operating with a terminal voltage of 250V and an internal generated voltage of 245V. Therefore, the armature current flow is $I_A = (250V\text{-}245V)/0.25\Omega = 20A$.

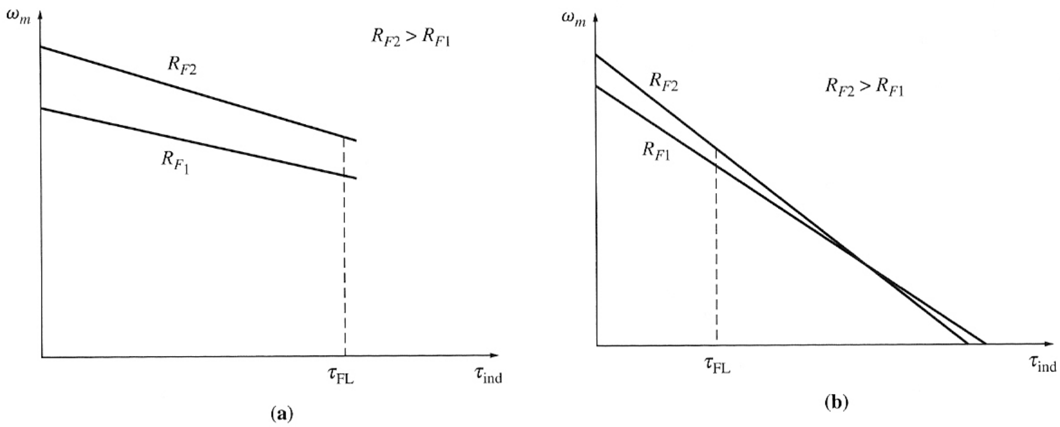
What happens in this motor if there is a 1% decrease in flux? If the flux decrease by 1%, then E_A must decrease by 1% too, because $E_A = K\phi\omega$. Therefore, E_A will drop to:
$E_{A2} = 0.99\ E_{A1} = 0.99\ (245) = 242.55V$

The armature current must then rise to
$I_A = (250\text{-}242.55)/0.25 = 29.8\ A$

Thus, a 1% decrease in flux produced a 49% increase in armature current.

So, to get back to the original discussion, the increase in current predominates over the decrease in flux. so, $\tau_{ind} > \tau_{load}$, the motor **speeds up**.

However, as the motor speeds up, E_A rises, causing I_A to fall. Thus, induced torque τ_{ind} drops too, and finally τ_{ind} equals τ_{load} at a higher steady-sate speed than originally.



The effect of field resistance R_F speed control on a shunt motor's torque-speed characteristics.
(a) over the motor's normal operating range
(b) over the entire range from no load to stall conditions

WARNING:

The effect of increasing the R_F is shown in figure (b) above. Notice that as the flux in the machine decreases, the no-load speed of the motor increases, while the slope of the torque-speed curve becomes steeper.

This figure shows the terminal characteristic of the motor over the full range from no-load to stall conditions. It is apparent that at very slow speeds an increase in field resistance will actually decrease the speed of the motor. This effect occurs because, at very low speeds, the increase in armature current caused by the decrease in E_A is no longer large enough to compensate for the decrease in flux in the induced torque equation. With the flux decrease actually larger than the armature current increase, the induced torque decreases, and the motor slows down.

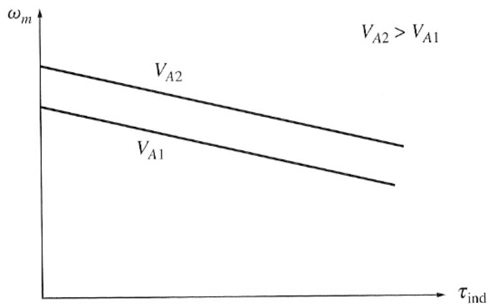
Some small dc motors used for control purposes actually operates at speeds close to stall conditions. For these motors, an increase in R_F might have no effect, or it might even decrease the speed of the motor. Since the results are not predictable, field resistance speed control should not be used in these types of dc motors. Instead, the armature voltage method should be employed.

Changing the Armature Voltage

This method involves changing the voltage applied to the armature of the motor without changing the voltage applied to the field.

If the voltage V_A is increased, then the I_A must rise [$I_A = (V_A \uparrow - E_A)/R_A$]. As I_A increases, the induced torque $\tau_{ind} = K\phi I_A\uparrow$ increases, making $\tau_{ind} > \tau_{load}$, and the speed of the motor increases.

But, as the speed increases, the E_A ($=K\phi\omega\uparrow$) increases, causing the armature current to decrease. This decrease in I_A decreases the induced torque, causing $\tau_{ind} = \tau_{load}$ at a higher rotational speed.



The effect of armature voltage speed control

Inserting a Resistor in Series with the Armature Circuit

If a resistor is inserted in series with the armature circuit, the effect is to drastically increase the slope of the motor's torque-speed characteristic, making it operate more slowly if loaded. This fact can be seen from the speed equation:

$$\omega = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2}\tau_{ind}$$

The insertion of a resistor is a very wasteful method of speed control, since the losses in the inserted resistor are very large. For this reason, it is rarely used.

Safe Ranges of Operation for the 2 common methods

Field Resistance Control

The lower the field current in a shunt (or separately excited) dc motor, the faster it turns: and the higher the field current, the slower it turns. Since an increase in field current causes decrease in speed, there is always a minimum achievable speed by field circuit control. This minimum speed occurs when the motor's field circuit has the maximum permissible current flowing through it.

If a motor is operating at its rated terminal voltage, power and field current, then it will be running at rated speed, also known as base speed. Field resistance control can control the speed of the motor for speeds above base speed but not for speeds below base speed. To achieve a speed slower than base speed by field circuit control would require excessive field current, possibly burning up the field windings.

Armature Voltage Control

The lower the armature voltage on a separately excited dc motor, the slower it turns, and the higher the armature voltage, the faster it turns. Since an increase in armature voltage causes an increase in speed, there is always a maximum achievable speed by armature voltage control. This maximum speed occurs when the motor's armature voltage reaches its maximum permissible level.

If a motor is operating at its rated terminal voltage, power and field current, then it will be running at rated speed, also known as base speed. Armature voltage control can control the speed of the motor for speeds below base speed but not for speeds above base speed. To achieve a speed faster than base speed by armature voltage control would require excessive armature voltage, possibly damaging the armature circuit.

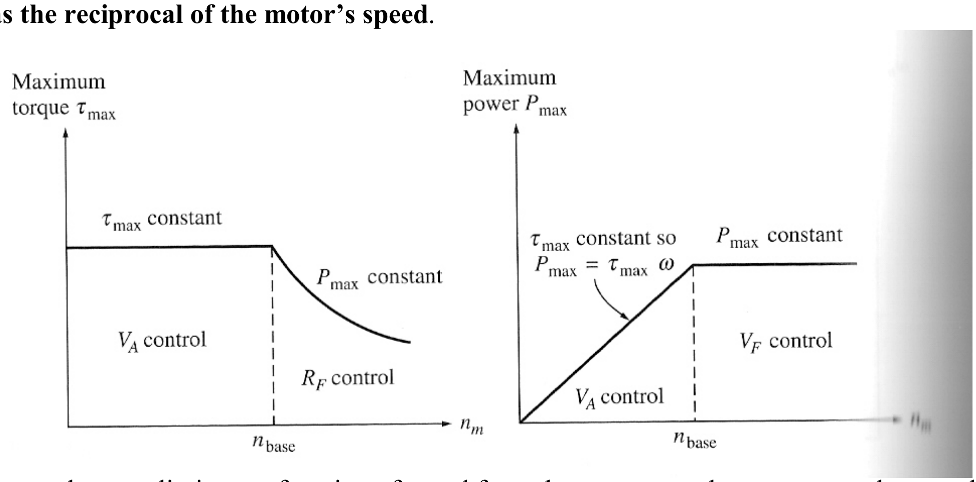
→ These two techniques of speed control are obviously complementary. Armature voltage control works well for speeds below base speed, and field resistance control works well for speeds above base speed.

There is a significant difference in the torque and power limits on the machine under these two types of speed control. The limiting factor in either case is the heating of the armature conductors, which places an upper limit on the magnitude of the armature current I_A.

For armature voltage control, the **flux in the motor is constant**, so the maximum torque in the motor is $\tau_{max} = K\phi I_{A,max}$

This maximum torque is constant regardless of the speed of the rotation of the motor. Since the power out of the motor is given by $P=\tau\omega$, the maximum power is $P_{max} = \tau_{max}\omega$. Thus, the **max power out is directly proportional to its operating speed under armature voltage control**.

On the other hand, when field resistance control is used, the flux does change. In this form of control, a speed increase is caused by a decrease in the machine's flux. In order for the armature current limit is not exceeded, the induced torque limit must decrease as the speed of the motor increases. Since the power out of the motor is given by $P=\tau\omega$ and the torque limit decreases as the speed of the motor increases, **the max power out of a dc motor under field current control is constant, while the maximum torque varies as the reciprocal of the motor's speed**.



Power and torque limits as a function of speed for a shunt motor under armature voltage and field resistance control

Example 9.3

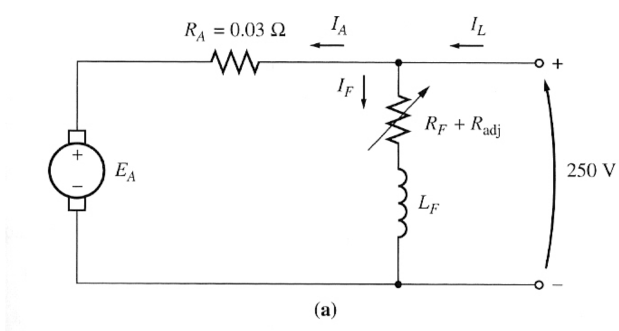
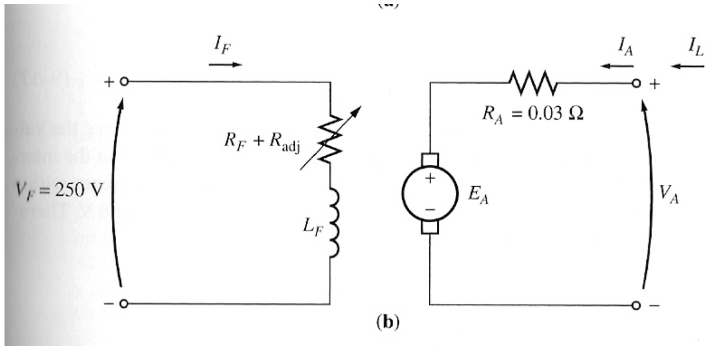


Figure above shows a 100hp, 250 V, 1200 r/min shunt dc motor with an armature resistance of 0.03 ohms and a field resistance of 41.67 ohms. The motor has compensating windings, so armature reaction can be ignored. Mechanical and core losses may be assumed to be negligible for the purposes of this problem. The motor is assumed to be driving a load with a line current of 126A and an initial speed of 1103 r/min. To simplify the problem, assume that the amount of armature current drawn by the motor remains constant.

(a) If the machine's magnetization curve is as in Example 9.2, what is the motor's speed if the field resistance is raised to 50 ohms?

Example 9.4

The motor in Example 9.3 is now connected separately excited as shown below. The motor is initially running with V_A = 250V, I_A = 120A, and n= 1103 r/min, while supplying a constant-torque load. What will the speed of this motor be if V_A is reduced to 200V?



The Effect of an Open Field Circuit

As the field resistance increased, the speed of the motor increased with it. What would happen if this effect were taken to the extreme, if the field resistance really increased? What would happen if the field circuit were actually opened while the motor is running?

From the previous discussion, the flux in the machine will drop, and E_A will drop as well. This would cause a really large increase in the armature current, and the resulting induced torque would be quite a bit higher than the load torque of the motor. Therefore, the motor's speed starts to rise and just keeps going up.

In ordinary shunt dc motors operating with light fields, if the armature reaction effects are severe enough, the effect of speed rising can take place. If the armature reaction on a dc motor is severe, an increase in load can weaken its flux enough to actually cause the motor's speed to rise. However, most loads have torque-speed curves whose torque increases with speed, so the increased speed increases its load, which increases the armature reaction, weakening the flux again. The weaker flux causes a further increase in speed, further increase the load, etc. etc. until the motor overspeeds. This condition is known as **runaway**.

4. The Permanent-Magnet DC Motor

A permanent magnet dc motor (PMDC) is a dc motor whose poles are made of permanent magnets. PMDC motor offer a number of benefits compared with shunt dc motors in some applications.

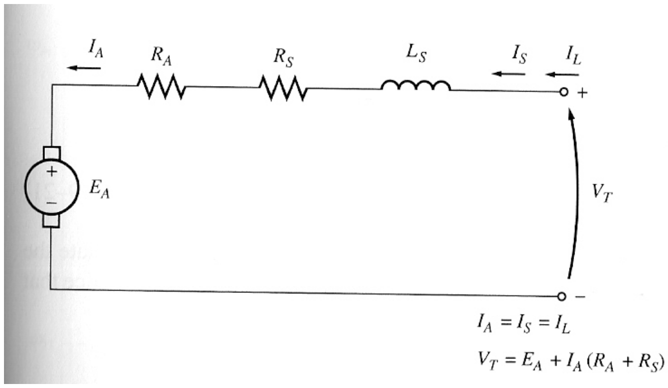
Advantage: Since the motors do not require an external field circuit, they do not have the field circuit copper losses. Because no field windings are required, they can be smaller than corresponding shunt dc motors.

Disadvantages:
Permanent magnets cannot produce as high flux density as an externally supplied shunt field., so a PMDC motor will have a lower induced torque per ampere of armature current than a shunt motor of the same size. Also, PMDC motors run the risk of demagnetization.

A PMDC motor is basically the same machine as a shunt dc motor, except that the flux of a PMDC motor is fixed. Therefore, it is not possible to control the speed of the PMDC motor by varying the field current or flux. The only methods of speed control available for a PMDC motor are armature voltage control and armature resistance control.

5. The Series DC Motor

A series DC motor is a dc motor whose field windings consist of relatively few turns connected in series with the armature circuit.



The KVL for this motor is $V_T = E_A + I_A (R_A + R_S)$

Induced Torque in a Series DC Motor

The basic behaviour of a series dc motor is due to the fact that the flux is directly proportional to the armature current, at least until saturation is reached. As the load on the motor increases, its flux increases too. As seen earlier, an increase in flux in the motor causes a decrease in its speed. The result is that a series dc motor has a sharply drooping torque-speed characteristic.

The induced torque is $\tau_{ind} = K\phi I_A$

The flux in this machine is directly proportional to its armature current (at least until metal saturates). Therefore, the flux in the machine can be given by $\phi = cI_A$ where c is a constant of proportionality. Thus, $\tau_{ind} = K\phi I_A = KcI_A^2$

Series dc motors are therefore used in applications requiring very high torques. Example: starter motors in cars, elevator motors, tractor motors etc.

The Terminal Characteristic of a Series DC Motor

The assumption of a linear magnetization curve implies that the flux in the motor will be given by $\phi = cI_A$. This equation will be used to derive the torque-speed characteristic curve for the series motor.

Derivation of the torque-speed characteristic:

1. $V_T = E_A + I_A (R_A + R_S)$

2. $I_A = \sqrt{\frac{\tau_{ind}}{Kc}}$

3. Also, $E_A = K\phi\omega$, thus substituting this in the KVL gives:

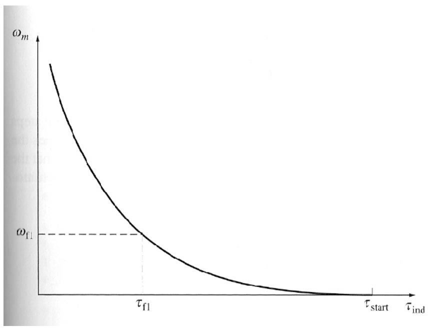
$$V_T = K\phi\omega + \sqrt{\frac{\tau_{ind}}{Kc}}(R_A + R_S)$$

4. If the flux can be eliminated from this expression, it will directly relate the torque of a motor to its speed. Notice that $I_A = \phi/c$ and $\tau_{ind} = (K/c)\phi^2$. Thus,

$$\phi = \sqrt{\frac{c}{K}}\sqrt{\tau_{ind}}$$

5. Substituting the flux equation into equation in part 3, results in:

$$\omega = \frac{V_T}{\sqrt{Kc}}\frac{1}{\sqrt{\tau_{ind}}} - \frac{R_A + R_S}{Kc}$$



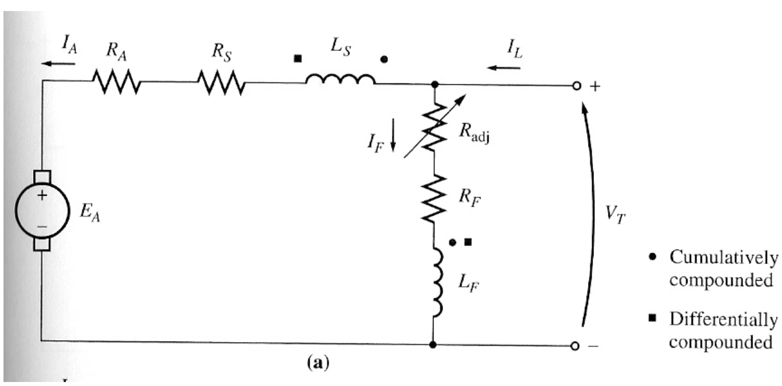
The torque-speed characteristic of a series dc motor

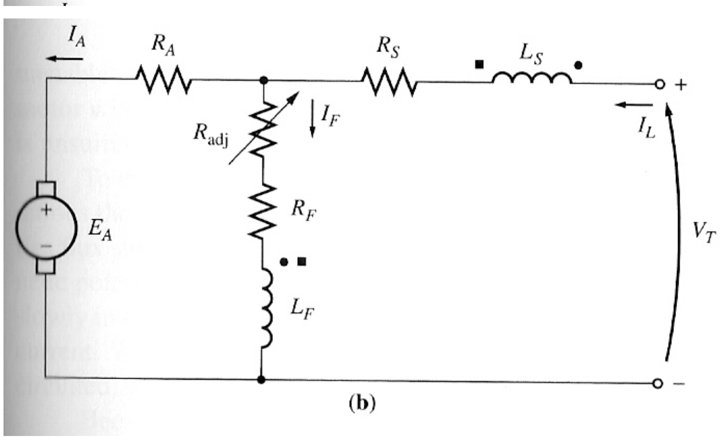
Speed Control of Series DC Motors.

Unlike with the shunt dc motor, there is only one efficient way to change the speed of a series dc motor. That method is to change the terminal voltage of the motor. If terminal voltage is increased, the speed will increase for any given torque.

6. The Compounded DC Motor

A compounded dc motor is a motor with both a shunt and a series field. This is shown below:





The equivalent circuit of compounded dc motor:
(a) long-shunt connection.
(b) short-shunt connection.

The KVL for a compounded dc motor is:

$$V_T = E_A + I_A (R_A + R_S)$$

and the currents are:

$$I_A = I_L - I_F$$
$$I_F = V_T/R_F$$

The net mmf and the effective shunt field current are:

$$F_{net} = F_F \pm F_{SE} - F_{AR}$$

$$I_F^* = I_F \pm (N_{SE}/N_F)\, I_A - F_{AR}/N_F$$

+ve sign associated with a cumulatively compounded motor
-ve sign associated with a differentially compounded motor

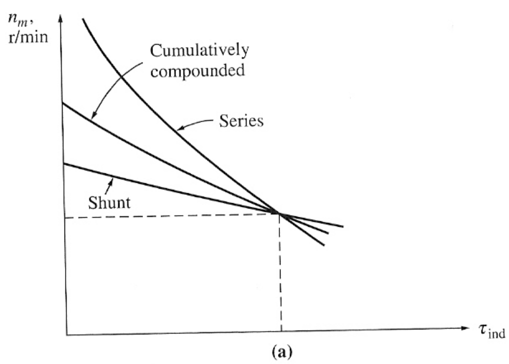
The Torque-Speed Characteristic of a Cumulatively Compounded DC Motor (CC)

There is a component of flux which is constant and another component which is proportional to its armature current (and thus to its load). Thus, CC motor has a higher starting torque than a shunt motor (whose flux is constant) but a lower starting torque than a series motor (whose entire flux is proportional to armature current).

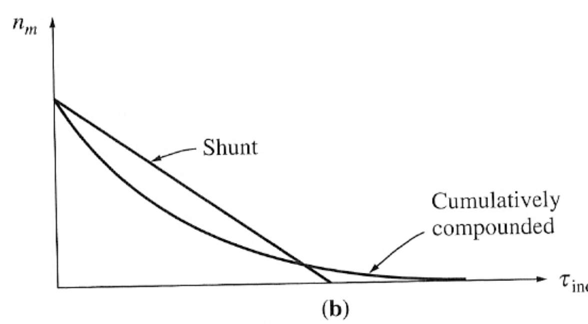
The CC motor combines the best features of both the shunt and series motors. Like a series motor, it has extra torque for starting; like a shunt, it does not overspeed at no load.

At light loads, the series field has a very small effect, so the motor behaves approximately as a shunt dc motor. As the load gets very large, the series flux becomes quite important and the torque-speed curve begins to look like a series motor's characteristic.

A comparison of the torque-speed characteristics of each of these types of machines is shown below:



The torque-speed characteristic of a cumulatively compounded dc motor compared to series and shunt motors with the same full-load rating.



The torque-speed characteristic of a cumulatively compounded dc motor compared to a shunt motor with the same no-load speed.

The Torque-Speed Characteristic of a Differentially Compounded DC Motor

In a differentially compounded dc motor, the shunt mmf and series mmf subtract from each other. This means that as the load on the motor increases, I_A increases and the flux in the motor decreases. But as the flux decreases, the speed of the motor increases. This speed increase causes another increase in load, which further increases I_A, further decreasing the flux, and increasing the speed again. The result is that a differentially compounded motor is unstable and tends to runaway.

This instability is much worse than that of a shunt motor with armature reaction. It is so bad that a differentially compounded motor is unsuitable for any application.

Differentially compounded motor is also impossible to start. At starting conditions, the armature current and the series field current are very high. Since the series flux subtracts from the shunt flux, the series field can actually reverse the magnetic polarity of the machine's poles. The motor will typically remain still or turn slowly in the wrong direction while burning up, because of the excessive armature current. When this type of motor is to be started, its series field must be short-circuited, so that it behaves as an ordinary shunt motor during the starting period.

Speed Control in the Cumulatively Compounded DC Motor

Same as for a shunt motor.